

# ALGEBRA

## SHEET 04

**Class notes by Aditya Ranjan**

# (Miscellaneous)



$$* (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$* (a+b)^2 - (a-b)^2 = 4ab$$

$$(123 = a) \quad (77 = b)$$

**1. Find the value of**

$$\frac{(a+b)^2 + (a-b)^2}{a^2 + b^2}$$
$$= \frac{2(a^2 + b^2)}{(a^2 + b^2)}$$

$$\frac{(123 + 77)^2 + (123 - 77)^2}{(123)^2 + (77)^2}$$

**(a) 0**

**(b) 1**

☒ **(c) 2**

**(d) 4**

**2. Find the value of**

$$\frac{(a+b)^2 - (a-b)^2}{ab}$$
$$= \frac{4ab}{ab}$$

$$\frac{(\overset{a}{148} + \overset{b}{69})^2 - (148 - 69)^2}{148 \times 69}$$

(a) 2

(c) 1

✓ (b) 4

(d) 0

# Rationalisation

$$* \frac{1}{\sqrt{3} + \sqrt{2}} = \sqrt{3} - \sqrt{2}$$

$$* \frac{1}{\sqrt{7} + \sqrt{6}} = \frac{\sqrt{7} - \sqrt{6}}{1}$$

$$* \frac{1}{\sqrt{5} + \sqrt{3}} = \frac{\sqrt{5} - \sqrt{3}}{2}$$

$$\frac{1}{\sqrt{2}+1} = \sqrt{2}-1$$

3.  $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \dots + \frac{1}{\sqrt{49}+\sqrt{48}}$

☒ (a) 6

(b) 8

(c) 7

(d) 0

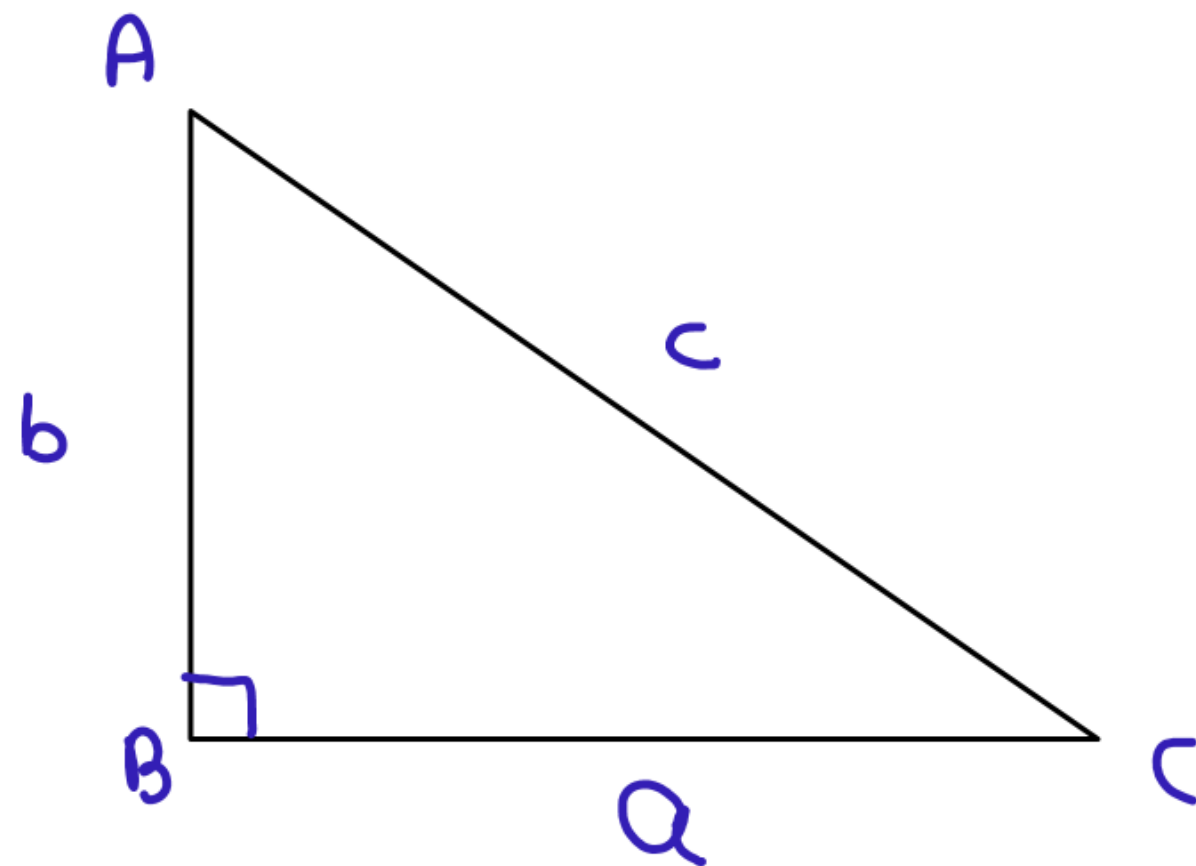
$$= \cancel{\sqrt{2}-1} + \cancel{\sqrt{3}-\sqrt{2}} + \dots + \cancel{\sqrt{48}-\sqrt{47}} + \sqrt{49}-\sqrt{48}$$

$$\begin{aligned} &= -1 + \sqrt{49} \\ &= -1 + 7 = 6 \end{aligned}$$

Pythagoras Theorem  $a^2 + b^2 = c^2$

Triplet  $\Rightarrow$

$\left\{ \begin{array}{l} 3, 4, 5 \\ 6, 8, 10 \\ 9, 12, 15 \\ 5, 12, 13 \\ 7, 24, 25 \\ 9, 40, 41 \end{array} \right.$



$$\sqrt[3]{x} = 2$$
$$\Rightarrow x = 8$$

4. If  $5^{\sqrt[3]{x}} + 12^{\sqrt[3]{x}} = 13^{\sqrt[3]{x}}$ , then the value of  $x$  is :

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(a) 2

(c) 1

(b) 8

(d) 4

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\sqrt[4]{x} = x^{\frac{1}{4}}$$

5. If  $6^{\sqrt[4]{x}} + 8^{\sqrt[4]{x}} = 10^{\sqrt[4]{x}}$ , then the value of  $x$  is :

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(a) 2

✓ (b) 16

(c) 4

(d) 8

$$\sqrt[4]{x} = 2$$

$$x^{\frac{1}{4}} = 2$$

$$\Rightarrow (x^{\frac{1}{4}})^4 = (2)^4$$

$$\Rightarrow x = 16$$

$$\checkmark * \sqrt{7+2\sqrt{6}} = \sqrt{6} + 1$$

$$\checkmark * \sqrt{8+4\sqrt{3}} = \sqrt{8+2\sqrt{2 \times 2 \times 3}} = \sqrt{8+2\sqrt{12}} = \sqrt{6} + \sqrt{2}$$

$$* \sqrt{4+2\sqrt{3}} = \sqrt{3} + 1$$

$$* \sqrt{2+\sqrt{3}} = \sqrt{\frac{4+2\sqrt{3}}{2}} = \frac{\sqrt{3}+1}{\sqrt{2}}$$

$$2\sqrt{3} = \sqrt{2 \times 2 \times 3} = \sqrt{12}$$

$$\sqrt{9} = \sqrt{3 \times 3} = (3^2)^{\frac{1}{2}} = 3$$





$$\sqrt{7} - \cancel{\sqrt{3}} + \sqrt{5} + \cancel{\sqrt{3}} = \sqrt{a} + \sqrt{b} \quad 8.$$

$$\sqrt{7} + \sqrt{5} = \sqrt{a} + \sqrt{b}$$

$$= \sqrt{7 \times 5} = \sqrt{35}$$

**If  $\sqrt{10 - 2\sqrt{21}} + \sqrt{8 + 2\sqrt{15}} = \sqrt{a} + \sqrt{b}$ , where  $a$  and  $b$  are positive integers, then the value of  $\sqrt{ab}$  is closest to :**

***CGL Tier-II (12 September 2019)***

**(a) 4.6**

**(c) 6.8**

☒ **(b) 5.9**

**(d)**







$$(a^4 + b^4 + a^2 b^2) = (a^2 + b^2 + ab)(a^2 + b^2 - ab)$$

$$91 = 13 \times 7$$

$$\textcircled{a} \quad a^2 + b^2 + ab = 13$$

$$a^2 + b^2 - ab = 7$$

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$$2a^2 + 2b^2 = 20$$

$$2(a^2 + b^2) = 20$$

$$\Rightarrow a^2 + b^2 = \frac{20}{2}$$

$$\textcircled{a} \quad a^2 + b^2 = \frac{20}{2} = 10$$

$$\textcircled{b} \quad ab = \frac{6}{2} = 3$$

$$\begin{aligned} \textcircled{c} \quad & \frac{1}{a^2} + \frac{1}{b^2} \\ &= \frac{b^2 + a^2}{(ab)^2} \\ &= \frac{10}{9} \end{aligned}$$

$$Q. \quad a^4 + b^4 + a^2b^2 = 90$$

$$a^2 + b^2 + ab = 18$$

$$a^2 + b^2 - ab = 5$$

$$(i) \quad a^2 + b^2 = \frac{23}{2}$$

$$(ii) \quad ab = \frac{13}{2}$$



12. If  $x^4 + x^2y^2 + y^4 = 273$  and  $x^2 - xy + y^2 = 13$ ,  
then the value of  $xy$  is :  $x^2 + xy + y^2 = 21$

*SSC CGL 5 March 2020 (Afternoon)*

(a) 4  
(c) 10

(b) 8  
(d) 6













$$13 = 10 \times 1 + 1 \times 3$$

$$453 = 100 \times 4 + 10 \times 5 + 3 \times 1$$

$$0.5 = 5 \times \frac{1}{10}$$

$$23.7 = 2 \times 10 + 3 \times 1 + 7 \times \frac{1}{10}$$



$$* a^m \times a^n = a^{m+n}$$

$$* \frac{a^m}{a^n} = a^{m-n}$$

$$* a^m = a^n$$

$$\Rightarrow m=n$$







21. If  $a^3 = 1$  and  $a^2 + a + 1 = 0$  then the value of  $a^5 + a^4 + 1$  is :  $= a^2 + a + 1$

- (a)  $a^2$
- (b) 1
- ✓ (c) 0
- (d)  $a + 1$







25. If  $x = \frac{\sqrt{3}}{2}$  then the value of  $\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}$  is

(a)  $-\sqrt{3}$

(b) 1

(c) -1

 (d)  $\sqrt{3}$















32. If  $ax + by = 4$ ,  $bx - ay = 3$ ,  $x^2 + y^2 = 5$  find  $a^2 + b^2 = ?$

(a) 15

(c) 35

☒ (b) 5

(d) None

$$(ax + by)^2 + (bx - ay)^2 = (a^2 + b^2)(x^2 + y^2)$$

$$4^2 + 3^2 = (a^2 + b^2) 5$$

$$\Rightarrow \frac{25}{5} = \cancel{5} (a^2 + b^2)$$

