TRIGONOMETRY

SHEET 07

MISCELLANEOUS

Questions Based on

2θ

$\sin 2\theta$

* $\sin 2\theta = 2\sin\theta \cdot \cos\theta$

* $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$\cos 2\theta$

$$\underline{\cos 2\theta} \rightarrow \begin{cases} 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta \\ \cos^2\theta - \sin^2\theta \quad \text{or} \quad \cos^4\theta - \sin^4\theta \\ \dfrac{1-\tan^2\theta}{1+\tan^2\theta} \end{cases}$$

Tan 2θ

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

1. If $2(\cos^2\theta - \sin^2\theta) = 1$ (θ is a positive acute angle), then $\cot\theta$ is equal to :

(a) $-\sqrt{3}$
(b) $\frac{1}{\sqrt{3}}$
(c) 1
(d) $\sqrt{3}$

$2 \times \cos 2\theta = 1$

$\Rightarrow \cos 2\theta = \frac{1}{2}$

$\Rightarrow \cos \underline{2\theta} = \cos \underline{60^\circ}$

$2\theta = 60^\circ$

$\theta = 30^\circ$

$\therefore \cot 30^\circ = \sqrt{3}$

2. The value of $\sec^2\theta - \dfrac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta}$ is :

(a) 1 (b) 2

(c) – 1 (d) 0

$= \sec^2\theta - \dfrac{\sin^2\theta\,\cancel{(1-2\sin^2\theta)}}{\cos^2\theta\,\cancel{(2\cos^2\theta-1)}}$

$= \sec^2\theta - \tan^2\theta$

$= 1$

$1-2\sin^2\alpha = \cos 2\alpha$

3. $1 - 2\sin^2\left(\frac{\pi}{4} + \theta\right) = ?$

(a) $\cos 2\theta$ (b) $-\cos 2\theta$

(c) $\sin 2\theta$ (d) $-\sin 2\theta$

$= \cos 2\left(\frac{\pi}{4} + \theta\right)$

$= \cos\left(\frac{\pi}{2} + 2\theta\right)$

$= -\sin 2\theta$

$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$

$= \cos\left(2 \times 22\frac{1}{2}^\circ\right)$

$= \cos 45^\circ$

$= \frac{1}{\sqrt{2}}$

4. Find the value of $\dfrac{1-\tan^2 22\frac{1}{2}^\circ}{1+\tan^2 22\frac{1}{2}^\circ}$ is :

(a) $\frac{\sqrt{3}}{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{2}$

(d) $\sqrt{3}$

$= \cos 30^\circ = \frac{\sqrt{3}}{2}$

5. The value of $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$ is :

(a) 1

(b) $\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$

(d) 2

$$\boxed{\sin 2\theta = 2\sin\theta \cdot \cos\theta}$$

6. $\sin 7\frac{1}{2}^\circ \sin 82\frac{1}{2}^\circ \cos 15^\circ = ?$

(a) $\frac{1}{2}$

(b) $\frac{1}{8}$

(c) $\frac{1}{4}$

(d) $\frac{1}{16}$

$= \sin 7\frac{1}{2}^\circ \cdot \cos 7\frac{1}{2}^\circ \cdot \cos 15^\circ$

$= \frac{2 \sin 7\frac{1}{2} \cdot \cos 7\frac{1}{2}}{2} \cdot \cos 15^\circ$

$= \frac{1}{2} \times \sin 15^\circ \cdot \cos 15^\circ$

$= \frac{1}{2} \times \frac{2}{2} \times \sin 15^\circ \cdot \cos 15$

$= \frac{1}{4} \sin 30^\circ = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

$= \left(\frac{1}{2}\right)^2 \cdot \sin 30^\circ$

$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

6. $\sin 7\frac{1}{2}^\circ \sin 82\frac{1}{2}^\circ \cos 15^\circ = ?$

(a) $\frac{1}{2}$

(b) $\frac{1}{8}$

(c) $\frac{1}{4}$

(d) $\frac{1}{16}$

* $x = \sin\theta \cdot \underbrace{\cos\theta \cdot \cos 2\theta \cdot \cos 4\theta}$.

$= \frac{1}{2} \times \sin 2\theta \cdot \cos 2\theta \cdot \cos 4\theta$

$= \frac{1}{4} \times \sin 4\theta \cdot \cos 4\theta \;=\; \frac{1}{8} \sin 8\theta$

$= \left(\frac{1}{2}\right)^3 \cdot \sin 8\theta$

* $x = \sin\alpha \cdot \cos\alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha$

$= \left(\frac{1}{2}\right)^4 \cdot \sin 16\alpha$

$= \frac{1}{16} \sin 16\alpha$

Q. $x = \sin 10^\circ \cdot \cos 10^\circ \cdot \cos 20^\circ \cdot \cos 40^\circ \cos 80^\circ$

$$= \left(\frac{1}{2}\right)^4 \times \sin 160^\circ$$

7. $\cot x - \tan x = ?$

(a) $2 \cot 2x$ (b) $2 \cot^2 x$

(c) $2 \cot^2 2x$ (d) $\cot^2 2x$

$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{2\times(\cos^2\theta - \sin^2\theta)}{2}}{\sin\theta \cdot \cos\theta} = \frac{2\times\cos 2\theta}{\sin 2\theta} = 2\cot 2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\Rightarrow 1 + \cos 2\theta = 2\cos^2\theta$

8. $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = ?$

(a) $\cos\theta$ (b) $\sin\theta$

(c) $2\cos\theta$ (d) $2\sin\theta$

$= \sqrt{2 + \sqrt{2(1 + \cos 4\theta)}}$

$= \sqrt{2 + \sqrt{2 \times 2\cos^2 2\theta}}$

$= \sqrt{2 + 2\cos 2\theta}$

$= \sqrt{2(1 + \cos 2\theta)} = \sqrt{2 \times 2\cos^2\theta} = 2\cos\theta$

9. If $\sin 2x = \frac{1}{5}$, then the value of $(\sin x + \cos x)$ is :

(a) $\sqrt{\frac{7}{5}}$ (b) $\sqrt{\frac{4}{5}}$

(c) $\sqrt{\frac{6}{5}}$ (d) $\sqrt{\frac{2}{5}}$

$1 + \sin 2x = \frac{1}{5} + 1$

$\Rightarrow \underbrace{\sin^2\theta + \cos^2\theta}_{} + \underbrace{2\sin\theta\cos\theta}_{} = \frac{6}{5}$

$\Rightarrow (\sin\theta + \cos\theta)^2 = \frac{6}{5}$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{\frac{6}{5}}$

* $\sin 2x = \frac{1}{5}$ $\qquad$ $\sin x - \cos x = ?$

$\Rightarrow \quad -\sin 2x = -\frac{1}{5}$

$\Rightarrow \quad 1 - \sin 2x = 1 - \frac{1}{5}$

$\Rightarrow \quad \sin^2 x + \cos^2 x - 2\sin x \cos x = \frac{4}{5}$

$\Rightarrow \quad (\sin x - \cos x)^2 = \frac{4}{5}$

$\Rightarrow \quad \sin x - \cos x = \sqrt{\frac{4}{5}}$ or $\frac{2}{\sqrt{5}}$

10. Find the value of $\tan\theta(1 + \sec 2\theta)(1 + \sec 4\theta)(1 + \sec 8\theta)$.

(a) $\tan 10\theta$
(b) $\tan 8\theta$
(c) $\tan 12\theta$
(d) 1

$= \tan\theta(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)$

$= \frac{\sin\theta}{\cos\theta}\left(1+\frac{1}{\cos 2\theta}\right)\left(1+\frac{1}{\cos 4\theta}\right)\left(1+\frac{1}{\cos 8\theta}\right)$

$= \frac{\sin\theta}{\cos\theta}\left(\frac{\cos 2\theta+1}{\cos 2\theta}\right)\left(\frac{\cos 4\theta+1}{\cos 4\theta}\right)\left(\frac{\cos 8\theta+1}{\cos 8\theta}\right)$

$= \frac{\sin\theta}{\cancel{\cos\theta}} \times \frac{2\cos^{\cancel{2}}\theta}{\cancel{\cos 2\theta}} \times \frac{2\cos^{\cancel{2}} 2\theta}{\cancel{\cos 4\theta}} \times \frac{2\cos^{\cancel{2}} 4\theta}{\cos 8\theta}$

$= \frac{\sin 2\theta \cdot 2\cos 2\theta \cdot 2\cos 4\theta}{\cos 8\theta}$

$= \frac{\sin 4\theta \cdot 2\cos 4\theta}{\cos 8\theta}$

$= \frac{\sin 8\theta}{\cos 8\theta}$

$= \tan 8\theta$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\Rightarrow \quad 2\sin^2\theta = 1 - \cos 2\theta$$

$$\Rightarrow \quad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\boxed{\Rightarrow \quad \sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}}$$

Ex:-

$$\sin 15^\circ = \sqrt{\frac{1-\cos 30^\circ}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\sin 22\tfrac{1}{2}^\circ = \sqrt{\frac{1-\cos 45^\circ}{2}}$$

$\sin 22\frac{1}{2}^\circ = \sqrt{\frac{1-\cos 45^\circ}{2}}$

$= \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}$

$= \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$

11. The value of $\sin 22\frac{1}{2}^\circ$ will be :

(a) $\sqrt{2}-1$

(b) $\frac{\sqrt{2}+1}{2\sqrt{2}}$

(c) $\frac{\sqrt{2}-1}{\sqrt{2}}$

(d) $\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$

Value Putting

$B = 45°$

$\tan A = \dfrac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \dfrac{\sqrt{2} - 1}{1}$

12. If $\tan A = \dfrac{1 - \cos B}{\sin B}$, then $\tan 2A$ is equal to :

(a) $\cot B$ 1 (b) $\tan B$ 1

(c) $\cos B$ $\frac{1}{\sqrt{2}}$ (d) $\operatorname{cosec} B$ $\sqrt{2}$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$= \dfrac{2(\sqrt{2}-1)}{1-(\sqrt{2}-1)^2} = \dfrac{2(\sqrt{2}-1)}{1-(2+1-2\sqrt{2})} = \dfrac{2(\sqrt{2}-1)}{1-3+2\sqrt{2}} = \dfrac{2(\sqrt{2}-1)}{2\sqrt{2}-2} = \dfrac{\cancel{2(\sqrt{2}-1)}}{\cancel{2(\sqrt{2}-1)}} = 1$

12. If $\tan A = \dfrac{1-\cos B}{\sin B}$, then $\tan 2A$ is equal to :

(a) $\cot B$
(b) $\tan B$
(c) $\cos B$
(d) $\operatorname{cosec} B$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$= \frac{2\times\left(\frac{1-\cos B}{\sin B}\right)}{1-\left(\frac{1-\cos B}{\sin B}\right)^2}$$

$$= \frac{\frac{2(1-\cos B)}{\sin B}}{\frac{\sin^2 B-(1-\cos B)^2}{\sin^2 B}} = \frac{\frac{2(1-\cos B)}{\cancel{\sin B}}}{\frac{(1-\cos^2 B)-(1-\cos B)^2}{\sin^{\cancel{2}} B}} = \frac{2\sin B}{(1+\cos B)-(1-\cos B)} = \frac{2\sin B}{\cancel{1}+\cos B-\cancel{1}+\cos B}$$

$$= \frac{\cancel{2}\sin B}{\cancel{2}\cos B}$$

Questions Based on

3θ

** $\boxed{3\theta}$

* $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

* $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

* $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

13. The value of $\frac{3\cos\theta + \cos 3\theta}{3\sin\theta - \sin 3\theta}$ is equal to :

(a) $\tan^3\theta$ (b) $\cot^3\theta$

(c) $\sin^3\theta$ (d) $\cos^3\theta$

$$= \frac{\cancel{3\cos\theta} + 4\cos^3\theta - \cancel{3\cos\theta}}{3\sin\theta - (3\sin\theta - 4\sin^3\theta)}$$

$$= \frac{\cancel{4}\cos^3\theta}{+\cancel{4}\sin^3\theta}$$

$$= \cot^3\theta$$

14. $\cos^2A(3 - 4\cos^2A)^2 + \sin^2A(3 - 4\sin^2A)^2$ is equal to :

(a) cos 4A (b) sin 4A

(c) 1 (d) None of these

$$\cos^2A\left[\frac{3\cos A - 4\cos^3A}{\cos A}\right]^2 + \sin^2A\left[\frac{3\sin A - 4\sin^3A}{\sin A}\right]^2$$

$$= \frac{\cancel{\cos^2A}}{\cancel{\cos^2A}}[-\cos 3A]^2 + \frac{\cancel{\sin^2A}}{\cancel{\sin^2A}}[\sin 3A]^2$$

$$= \cos^2 3A + \sin^2 3A$$

$$= 1$$

15. If $\tan A = \frac{1}{2}$, then $\tan 3A = ?$

(a) $\frac{9}{2}$

(b) $\frac{11}{2}$

(c) $\frac{7}{2}$

(d) $\frac{3}{2}$

$$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

$$= \frac{3 \times \frac{1}{2} - \left(\frac{1}{2}\right)^3}{1 - 3 \times \left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{3}{2} - \frac{1}{8}}{1 - 3 \times \frac{1}{4}}$$

$$= \frac{12-1}{8-6} = \frac{11}{2}$$

16. If $\sin x = \frac{2}{3}$, then find the value of $\cos 3x$.

SSC CHSL 15/10/2020 (Morning)

(a) 0.6735 (b) – 0.8765

(c) – 0.5797 (d) 0.5678

$\cos 3x = 4\cos^3 x - 3\cos x$

$= 4 \times \left(\frac{\sqrt{5}}{3}\right)^3 - \cancel{3} \times \frac{\sqrt{5}}{\cancel{3}}$

$= \frac{4 \times 5\sqrt{5}}{27} - \sqrt{5}$

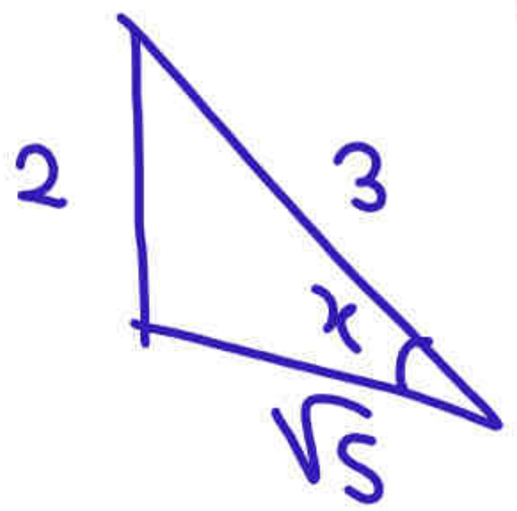
$= \sqrt{5}\left(\frac{20}{27} - 1\right)$

$= \sqrt{5}\left(-\frac{7}{27}\right)$

$= -(2.23) \times 0.25$

$= -0.5575$

27) 70 (.25
54
160
135



$\sqrt{2} = 1.414$
$\sqrt{3} = 1.732$
$\sqrt{5} = 2.236$

Miscellaneous

17. If $\tan A + \cot A = 2$, then the value of $\tan^{10}A + \cot^{10}A$ is :

(a) 4 (b) 2

(c) 2^{10} (d) 1

$\tan A + \frac{1}{\tan A} = 2$

$\boxed{\therefore \tan A = 1}$

then $\tan^{10}A + \frac{1}{\tan^{10}A}$

$= 1 + 1$

$= 2$

(a) $\tan^{10}A + \cot^{10}A = 2$

(b) $\tan^{20}A + \cot^{20}A = 2$

18. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^n\theta + \cot^n\theta$ $(0^\circ \le \theta \le 90^\circ)$ is equal to :

(a) 2
(b) 2^n
(c) $2n$
(d) 2^{n+1}

* $\tan A + \frac{1}{\tan A} = 2$ $\qquad$ $\tan A = 1$

* $\sin A + \frac{1}{\sin A} = 2$ $\qquad$ $\sin A = 1$

* $\cos A + \frac{1}{\cos A} = 2$ $\qquad$ $\cos A = 1$

19. If $\sin\theta + \text{cosec}\theta = 2$, then the value of $\sin^{100}\theta + \text{cosec}^{100}\theta$ is equal to :

(a) 1 (b) 2

(c) 3 (d) 100

$\sin\theta + \frac{1}{\sin\theta} = 2$

$\sin\theta = 1$

20. If $\sin\theta + \operatorname{cosec}\theta = 2$, then what is the value of $(\sin^{153}\theta + \operatorname{cosec}^{253}\theta)$?

SSC CHSL 10 July 2019 (Afternoon)

(a) $\frac{1}{153 \times 253}$

(b) $\frac{253}{123}$

(c) 2

(d) $\frac{153}{253}$

21. If $\cos\theta + \sec\theta = 2$, then the value of $\cos^6\theta + \sec^6\theta$ is :

(a) 4 (b) 8

(c) 1 (d) 2

$\cos\theta + \frac{1}{\cos\theta} = 2$ $\cos\theta = 1$

$\cos^6\theta + \frac{1}{\cos^6\theta}$

22. If $\sec x + \cos x = 2$, then the value of $\sec^{16}x + \cos^{16}x$ will be :

(a) $\sqrt{3}$

(b) 2

(c) 1

(d) 0

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (3x)^2 - \left(\frac{3}{x}\right)^2 = 1$

$\Rightarrow 9x^2 - \frac{9}{x^2} = 1$

$\Rightarrow 9\left(x^2 - \frac{1}{x^2}\right) = 1$

23. If $\sec\theta = 3x$ and $\tan\theta = \frac{3}{x}$, $(x \neq 0)$, then the value of $9\left(x^2 - \frac{1}{x^2}\right)$ is :

SSC CHSL 5 July 2019 (Evening)

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) 1

(d) $\frac{1}{4}$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \quad 9x^2 - \frac{9}{x^2} = 1$

$\Rightarrow \quad 6\times\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{9}\times 6 \quad \frac{2}{3}$

24. If $\operatorname{cosec}\theta = 3x$ and $\cot\theta = \frac{3}{x}$, $(x \neq 0)$, then the value of $6\left(x^2 - \frac{1}{x^2}\right)$ is :

SSC CHSL 8 July 2019 (Morning)

(a) $\frac{2}{3}$ (b) 1

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

25. If sec x + cos x = 3, then the value of $\tan^2 x - \sin^2 x$ is :

(a) 5 (b) 13

(c) 9 (d) 4

$\sec^2 x + \cos^2 x + 2 \times \cancel{\sec x} \times \cancel{\cos x} = 9$

$\Rightarrow 1 + \tan^2 x + 1 - \sin^2 x + 2 = 9$

$\Rightarrow \tan^2 x - \sin^2 x = 5$

$\sin\theta + \sin^2\theta = 1$

$\Rightarrow \sin\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta = \cos^2\theta$

26. If $\sin\theta + \sin^2\theta = 1$, then find the value of $\cos^2\theta + \cos^4\theta$.

(a) 1 (b) 5

(c) 0 (d) – 1

$\cos^2\theta + \cos^4\theta$

$= \sin\theta + \sin^2\theta$

$= 1$

$\cos\theta + \cos^2\theta = 1$

$\Rightarrow \cos\theta = \sin^2\theta$

27. If $\cos\theta + \cos^2\theta = 1$, then find the value of $\sin^2\theta + \sin^4\theta$.

(a) 1
(b) 5
(c) 0
(d) – 1

$\cos\theta + \cos^2\theta = 1$

$$(a+b)^2 = a^2 + 2ab + b^2$$

1 2 1

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

1 3 3 1

$\cos A + \cos^2 A = 1$

$\Rightarrow \cos A = \sin^2 A$

28. If $\cos A + \cos^2 A = 1$, then find the value of $\sin^{12}A + 3\sin^{10}A + 3\sin^8 A + \sin^6 A + \sin^4 A + \sin^2 A$ is :

(a) -1 (b) 5

(c) 2 (d) 1

$\underbrace{\cos^6 A + 3\cos^5 A + 3\cos^4 A + \cos^3 A} + \cos^2 A + \cos A$

$= (\cos^2 A + \cos A)^3 + (\cos^2 A + \cos A)$

$= 1 + 1$

29. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, then $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta$ is equal to :

(a) 2 (b) 1

(c) 4 (d) 3

$\sin\theta + \sin^2\theta + \sin^3\theta = 1$

$\Rightarrow \sin\theta + \sin^3\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta(1 + \sin^2\theta) = \cos^2\theta$

$\Rightarrow \sin^2\theta(1 + 1 - \cos^2\theta)^2 = \cos^4\theta$

$\Rightarrow (1 - \cos^2\theta)(2 - \cos^2\theta)^2 = \cos^4\theta$

$\Rightarrow (1 - \cos^2\theta)(4 + \cos^4\theta - 4\cos^2\theta) = \cos^4\theta$

$\Rightarrow 4 + \cancel{\cos^4\theta} - 4\cos^2\theta - 4\cos^2\theta - \cos^6\theta + 4\cos^4\theta = \cancel{\cos^4\theta}$

$\boxed{\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4}$

30. If $\theta = 60°$, then $\frac{1}{2}\sqrt{1+\sin\theta} + \frac{1}{2}\sqrt{1-\sin\theta}$ is equal to :

(a) $\cot\frac{\theta}{2}$ (b) $\sec\frac{\theta}{2}$

(c) $\sin\frac{\theta}{2}$ (d) $\cos\frac{\theta}{2}$ ✓

$= \frac{1}{2}\left(\sqrt{1+\frac{\sqrt{3}}{2}} + \sqrt{1-\frac{\sqrt{3}}{2}}\right)$

$= \frac{1}{2}\left(\sqrt{\frac{2+\sqrt{3}}{2}} + \sqrt{\frac{2-\sqrt{3}}{2}}\right)$

$= \frac{1}{2}\left(\sqrt{\frac{4+2\sqrt{3}}{4}} + \sqrt{\frac{4-2\sqrt{3}}{4}}\right)$

$= \frac{1}{2}\left(\frac{\sqrt{3}+1}{2} + \frac{\sqrt{3}-1}{2}\right)$

$= \frac{1}{2}\left(\frac{\sqrt{3}+\cancel{1}+\sqrt{3}-\cancel{1}}{2}\right)$

$= \frac{1}{2} \times \frac{\cancel{2}\sqrt{3}}{\cancel{2}} = \frac{\sqrt{3}}{2} = \cos 30° = \cos\frac{\theta}{2}$

* $\sin\theta \cdot \sin(60-\theta) \cdot \sin(60+\theta) = \frac{1}{4}\sin 3\theta$
* $\cos\theta \cdot \cos(60-\theta) \cdot \cos(60+\theta) = \frac{1}{4}\cos 3\theta$
* $\tan\theta \ \tan(60-\theta) \cdot \tan(60+\theta) = \tan 3\theta$.

31. Find $\sin 20^\circ . \sin 40^\circ . \sin 60^\circ . \sin 80^\circ$.

$\theta \quad 60-\theta \quad 60+\theta$

(a) $\frac{1}{16}$

(b) $\frac{5}{16}$

(c) $\frac{3}{16}$ ✓

(d) $\frac{1}{8}$

$= \sin 20^\circ . \sin 40^\circ . \sin 80^\circ . \sin 60^\circ$

$= \frac{1}{4} \sin 60^\circ . \sin 60^\circ$

$= \frac{1}{4} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$

32. Find $1 - \sin 10^\circ . \sin 50^\circ . \sin 70^\circ$.

(a) $\frac{5}{8}$

(b) $\frac{7}{8}$

(c) $\frac{3}{8}$

(d) $\frac{1}{8}$

$1 - \sin 10^\circ \cdot \sin 50 \cdot \sin 70^\circ$

$= 1 - \sin 10^\circ \cdot \sin(60 - 10^\circ) \cdot \sin(60 + 10^\circ)$

$= 1 - \frac{1}{4} \sin 30^\circ$

$= 1 - \frac{1}{4} \times \frac{1}{2} = 1 - \frac{1}{8} = \frac{7}{8}$

33. Find sin12°.sin48°.sin54°.

(a) $\frac{5}{8}$

(b) $\frac{7}{8}$

(c) $\frac{3}{8}$

(d) $\frac{1}{8}$

$= \sin 12^\circ \cdot \sin 48^\circ \cdot \frac{\sin 72^\circ}{\sin 72^\circ} \cdot \sin 54^\circ$

$= \frac{1}{4} \cdot \frac{\sin 36^\circ}{\sin 72^\circ} \cdot \sin 54^\circ$

$= \frac{1}{4} \times \frac{\cancel{\sin 36^\circ} \cdot \cos \cancel{36}}{2 \cancel{\sin 36} \cdot \cos \cancel{36}^\circ}$

$= \frac{1}{8}$

34. **Find sin6°.sin42°.sin66°.sin78°.**

(a) $\frac{1}{16}$ (b) $\frac{5}{16}$

(c) $\frac{3}{16}$ (d) $\frac{1}{8}$

$= \frac{\sin 6^\circ \times \sin 54 \times \sin 66^\circ}{\sin 54^\circ} \times \sin 42^\circ \times \sin 78^\circ$

$= \frac{1}{4} \frac{\sin 18 \times \sin 42^\circ \times \sin 78^\circ}{\sin 54^\circ}$

$= \frac{1}{4}\left[\frac{1}{4} \cdot \frac{\cancel{\sin 54}}{\cancel{\sin 54}}\right]$

$= \frac{1}{16}$

35. sin12°.sin24°.sin48°.sin84° = ?

(a) cos20°.cos40°.cos60°.cos80°

(b) sin20°.sin40°.sin60°.sin80°

(c) $\frac{3}{16}$

(d) $\frac{1}{16}$

$$\frac{\sin 12^\circ \cdot \sin 48^\circ \cdot \sin 72^\circ \cdot \sin 24^\circ \cdot \sin 84^\circ}{\sin 72^\circ}$$

$$= \frac{1}{4} \cdot \frac{\overset{60-\theta}{\sin 36^\circ} \cdot \overset{\theta}{\sin 24} \cdot \overset{60+\theta}{\sin 84^\circ}}{\sin 72^\circ}$$

$$= \frac{1}{4}\left[\frac{1}{4}\frac{\sin \cancel{72}}{\sin \cancel{72^\circ}}\right]$$

$$= \frac{1}{16}$$

$= \frac{1}{4} \cos 45°$

$= \frac{1}{4} \times \frac{1}{\sqrt{2}}$

36. The value of $(\cos 15°.\cos 45°.\cos 75°)$ is :

(a) $\frac{1}{3\sqrt{2}}$

(b) $\frac{1}{4\sqrt{2}}$

(c) $\frac{1}{8}$

(d) $\frac{\sqrt{3}}{8}$

$$x = \cos 10^\circ \cdot \cos 20^\circ \cos 40^\circ$$

$$= \frac{2}{2} \times \frac{\sin 10 \cdot \cos 10}{\sin 10} \cos 20 \cdot \cos 40$$

$$= \frac{\boxed{2\times \sin 20^\circ \cdot \cos 20^\circ}\ \cos 40^\circ}{2\times\ 2\sin 10^\circ}$$

$$= \frac{2\times}{2\times} \frac{\sin 40^\circ \cdot \cos 40^\circ}{4 \sin 10^\circ}$$

$$= \frac{\sin 80^\circ}{8 \sin 10^\circ} = \frac{\cos 10^\circ}{8 \sin 10^\circ} = \frac{1}{8} \cot 10^\circ$$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

37. If $x = \cos 10^\circ . \cos 20^\circ . \cos 40^\circ$, then $x = ?$

(a) $\frac{1}{4}\tan 10^\circ$ (b) $\frac{1}{8}\tan 10^\circ$

(c) $\frac{1}{4}\cot 10^\circ$ (d) $\frac{1}{8}\cot 10^\circ$

$x = \cos 10 \cdot \cos 20 \cdot \cos 40^\circ$

$= \cos 10^\circ \frac{\cos 20^\circ \cos 40^\circ . \cos 80^\circ}{\cos 80^\circ}$

$= \frac{\cos 10^\circ}{\cos 80} \times \frac{1}{4} \cdot \cos 60^\circ$

$= \frac{\cos 10^\circ}{\sin 10^\circ} \times \frac{1}{4} \times \frac{1}{2}$

$= \frac{1}{8}\cot 10^\circ$

$= \tan 60^\circ$

$= \sqrt{3}$

38. The value of $(\tan 20^\circ . \tan 40^\circ . \tan 80^\circ)$ is :

(a) 1 (b) 0

(c) $\sqrt{3}$ (d) 3

$* \tan\theta \cdot \tan(60-\theta) \tan(60+\theta) = \tan 3\theta$

39. The value of $\tan 6^\circ . \tan 42^\circ . \tan 66^\circ . \tan 78^\circ$ is :

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

$$= \frac{\tan 6^\circ \cdot \tan 54^\circ \cdot \tan 66^\circ}{\tan 54^\circ} \tan 42^\circ \tan 78^\circ$$

$$= \frac{\tan 18^\circ \tan 42^\circ \tan 78^\circ}{\tan 54^\circ}$$

$$= \frac{\cancel{\tan 54}}{\cancel{\tan 54}}$$

40. $\cot\theta \times \cot(60^\circ - \theta) \times \cot(60^\circ + \theta) = ?$

(a) $\cot 2\theta$ (b) $\cot 3\theta$

(c) $\cot\theta$ (d) $\cot 4\theta$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\Rightarrow \boxed{\tan(45 + \theta)} = \frac{\tan 45^\circ + \tan \theta}{1 - \tan 45^\circ \cdot \tan \theta}$$

$$= \frac{1 + \tan \theta}{1 - \tan \theta}$$

* $$\frac{1+\tan\theta}{1-\tan\theta} = \tan(45+\theta)$$

* $$\frac{1-\tan\theta}{1+\tan\theta} = \tan(45-\theta)$$

$$\frac{1+\tan 17^\circ}{1-\tan 17^\circ}$$

$= \tan(45+17^\circ)$

$= \tan 62^\circ$

41. The value of $\dfrac{\cos 17^\circ + \sin 17^\circ}{\cos 17^\circ - \sin 17^\circ}$ is :

(a) $\tan 17^\circ$
(b) $\tan 34^\circ$
(c) $\tan 62^\circ$
(d) $\tan 73^\circ$

42. If $\tan\theta = \dfrac{\cos 12^\circ - \sin 12^\circ}{\cos 12^\circ + \sin 12^\circ}$ and $(0^\circ \le \theta < 90^\circ)$, then θ is equal to :

(a) 48° (b) 24°

(c) 33° (d) 30°

$\tan\theta = \dfrac{1 - \tan 12}{1 + \tan 12^\circ}$

$\Rightarrow \tan\theta = \tan(45^\circ - 12^\circ)$

$\Rightarrow \tan\theta = \tan 33^\circ$

* $\sin(x+y) = \sin x \cos y + \cos x \sin y$
* $\sin(x-y) = \sin x \cos y - \cos x \sin y$
* $\cos(x+y) = \cos x \cos y - \sin x \sin y$
* $\cos(x-y) = \cos x \cos y + \sin x \sin y$
* $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$
* $\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

43. $\dfrac{\sin(B+A)+\cos(B-A)}{\sin(B-A)-\cos(B+A)} = ?$

(a) $\dfrac{\cos B+\sin B}{\sin B-\cos B}$

(b) $\dfrac{\cos A-\sin A}{\cos A+\sin A}$

(c) $\dfrac{\cos A+\sin A}{\cos A-\sin A}$

(d) $\dfrac{\cos B-\sin B}{\sin B+\sin B}$

$$= \frac{\sin B\cdot\cos A+\cos B\sin A+\cos B\cos A+\sin B\sin A}{\sin B\cos A-\cos B\sin A-[\cos A\cos B-\sin A\sin B]}$$

$$= \frac{\sin B[\cos A+\sin A]+\cos B[\sin A+\cos A]}{\sin B[\cos A+\sin A]-\cos B[\sin A+\cos A]}$$

$$= \frac{\cancel{[\sin A+\cos A]}[\sin B+\cos B]}{\cancel{[\sin A+\cos A]}[\sin B-\cos B]}$$

44. If $\tan(A - B) = x$, then the value of x is :

(a) $\dfrac{\tan A + \tan B}{1 - \tan A.\tan B}$ (b) $\dfrac{\tan A + \tan B}{1 + \tan A.\tan B}$

(c) $\dfrac{\tan A - \tan B}{1 - \tan A.\tan B}$ (d) $\dfrac{\tan A - \tan B}{1 + \tan A.\tan B}$

45. Find $\dfrac{\tan\left(\dfrac{\pi}{4}+x\right)}{\tan\left(\dfrac{\pi}{4}-x\right)} = ?$

(a) $\left[\dfrac{1-\tan x}{1+\tan x}\right]^2$

(b) $\left[\dfrac{1+\tan x}{1+\tan x}\right]^2$

(c) $\left[\dfrac{1-\tan x}{1-\tan x}\right]^2$

(d) $\left[\dfrac{1+\tan x}{1-\tan x}\right]^2$

$= \dfrac{\tan(45+x)}{\tan(45-x)}$

$= \dfrac{\dfrac{1+\tan x}{1-\tan x}}{\dfrac{1-\tan x}{1+\tan x}} = \dfrac{(1+\tan x)^2}{(1-\tan x)^2}$

$A + C = B$

$\tan(A+C) = \tan B$

$\Rightarrow \dfrac{\tan A + \tan C}{1 - \tan A \tan C} = \tan B$

46. If A + C = B, then tanA.tanB.tanC = ?

(a) tanA.tanB – tanC
(b) tanA – tanB – tanC
(c) tanB – tanA – tanC
(d) tanC – tanA.tanB

$\Rightarrow \tan A + \tan C = \tan B - \tan A \tan B \tan C$

$\Rightarrow \boxed{\tan A \tan B \tan C = \tan B - \tan A - \tan C}$

47. If A, B, C are angles of a triangle, then

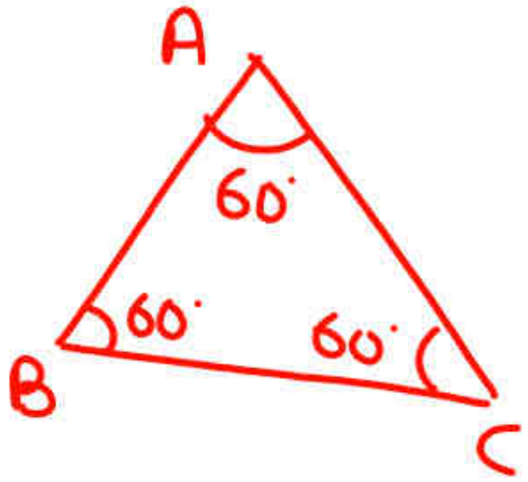
$$\frac{\tan A + \tan B + \tan C}{\tan A . \tan B . \tan C} = ?$$

(a) $\frac{1}{\sqrt{3}}$ (b) 2

(c) 1 (d) $\sqrt{3}$

$= \frac{3\cancel{\tan A}}{\cancel{\tan^3 A}}$ $\tan^2 A$

$= \frac{\cancel{3}}{\cancel{3}} = 1$



* If $x+y=z$ find $\tan x \cdot \tan y \cdot \tan z$

$= \tan z - \tan x - \tan y$ ✗

* If $\theta + 2\theta = 3\theta$ find $\tan\theta \cdot \tan 2\theta \cdot \tan 3\theta$

$= \tan 3\theta - \tan\theta - \tan 2\theta$

$x + 2x = 3x$

48. Find $\tan 3x \tan 2x \tan x$.

(a) $\tan 3x - \tan 2x + \tan x$

(b) $\tan 3x + \tan 2x - \tan x$

(c) $\tan 3x - \tan 2x - \tan x$

(d) $\tan 3x + \tan 2x + \tan x$

48. **Find $\tan 3x \tan 2x \tan x$.**

(a) $\tan 3x - \tan 2x + \tan x$

(b) $\tan 3x + \tan 2x - \tan x$

(c) $\tan 3x - \tan 2x - \tan x$

(d) $\tan 3x + \tan 2x + \tan x$

$x + 2x = 3x$

$\Rightarrow \tan(x + 2x) = \tan 3x$

$\Rightarrow \dfrac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = \tan 3x$

$\Rightarrow \tan x + \tan 2x = \tan 3x - \underline{\tan x \tan 2x \tan 3x}$

$\Rightarrow \tan x \tan 2x \tan 3x = \tan 3x - \tan x - \tan 2x$

$2x + 3x = 5x$

49. $\tan 5x . \tan 3x . \tan 2x =$

(a) $\tan 5x - \tan 3x - \tan 2x$

(b) $\dfrac{\sin x - \sin 3x - \sin 2x}{\cos 5x - \cos 3x - \cos 2x}$

(c) 0

(d) $\tan 5x + \tan 3x + \tan 2x$

V.V. Imp

50. If A + B = 45°, then (1 + tanA)(1 + tanB) is equal to :

45° 0

(a) 2 (b) 1

(c) 0 (d) 4

$(1+\tan A)(1+\tan B)$

$= (1+1)(1+0)$

$= 2\times 1 = 2$

If $A+B=45°$

$(1+\tan A)(1+\tan B)=2$

$A - B = 45^\circ$

$45 - 0 = 45^\circ$

$(1+\tan A)(1-\tan B)$

$= (1+1)(1-0)$

$= 2 \times 1 = 2$

51. If $A - B = \frac{\pi}{4}$, then $(1 + \tan A)(1 - \tan B) = ?$

(a) 1 (b) 2

(c) – 1 (d) – 2

Q. $A+B = 135^\circ$ $\quad (1+\cot A)(1+\cot B) = ?$

$$\underbrace{A}_{90} + \underbrace{B}_{45^\circ} = 135^\circ$$

$$(1+\cot A)(1+\cot B)$$
$$= (1+\cot 90^\circ)(1+\cot 45^\circ)$$
$$= (1+0)(1+1)$$
$$= 1 \times 2 = 2$$

* $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

* $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

* $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$

* $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

52. Find $\dfrac{\cos 7x + \cos 5x}{\sin 7x - \sin 5x} = ?$

(a) $\cot 2x$ (b) $\cot 3x$

(c) $\cot x$ (d) $\cot 6x$

2017 Mains

$$= \frac{\cancel{2}\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)}{\cancel{2}\cos\left(\frac{7x+5x}{2}\right)\cdot\sin\left(\frac{7x-5x}{2}\right)}$$

$$= \frac{\cancel{\cos 6x}\cdot\cos x}{\cancel{\cos 6x}\cdot\sin x}$$

$\sin 75 + \sin 15$

$= 2\sin\left(\frac{75+15}{2}\right)\cdot\cos\left(\frac{75-15}{2}\right)$

$= 2\sin 45^\circ \cdot \cos 30^\circ$

$= \cancel{2} \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{\cancel{2}} = \sqrt{\frac{3}{2}}$

53. What is the value of sin75° + sin15° ?

2017 Mains

(a) $\sqrt{3}$

(b) $\frac{2}{\sqrt{3}}$

(c) $\sqrt{\frac{3}{2}}$ ✓

(d) $\frac{3}{\sqrt{2}}$