TRIGONOMETRY

MAX./ MIN.

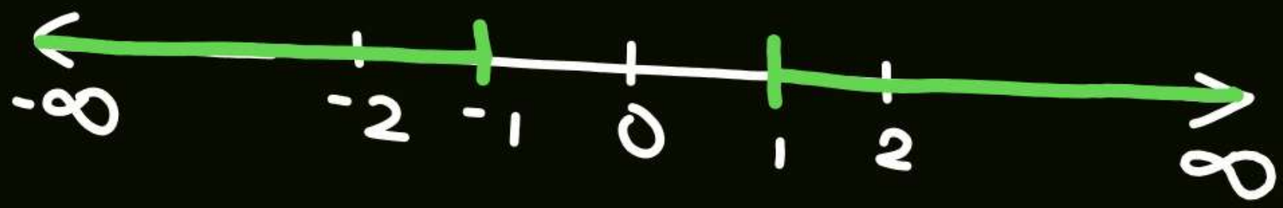
Class Notes by Aditya Ranjan Sir

0° 30° 45° 60° 90° -

$\cos 0^\circ = 1$

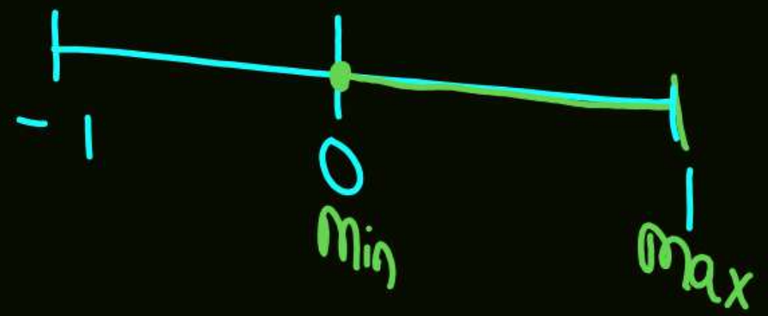
$\cos 180^\circ = -1$

	Min^m	Max^m
$\sin\theta/\cos\theta$	-1	1
$\tan\theta/\cot\theta$	$-\infty$	∞
$\sec\theta$	$(-\infty,-1]\cup[1,\infty)$	



	Min	Max
* $\sin\theta$	-1	1
* $2\sin\theta$	-2	2
* $7\sin\theta$	-7	7
* $4\cos\theta$	-4	4

	Min	Max^m
* $\sin\theta$	-1	$+1$
* $\sin^2\theta$	0	1
* $\sin^3\theta$	-1	1
* $7\sin^2\theta$	0	7
* $8\sin^3\theta$	-8	8



Min Max

* $\sin\theta$
* $\sin 2\theta$
* $\sin 3\theta$
* $7\sin 3\theta$
* $8\sin^3 7\theta$

$\sin\theta + \sin\theta$

$\sin\theta$		$\sin 2\theta$		$2\sin\theta$	
-1	1	-1	1	-2	2

	Min	Max
* $\sin\theta$	-1	1
* $\sin 2\theta$	-1	1
* $\sin 3\theta$	-1	1
* $7\sin 3\theta$	-7	7
* $8\sin^3 7\theta$	-8	8
* $4\sin^2 2\theta$	0	4

Min-Max table

	Min value	Max value	Can be written as
$\sin\theta, \sin2\theta, \sin9\theta \ldots \sin n\theta$	-1	$+1$	$-1 \le \sin n\theta \le 1$
$\cos\theta, \cos4\theta, \cos7\theta \ldots \cos n\theta$			$-1 \le \cos n\theta \le 1$
$\sin^2\theta, \sin^24\theta, \sin^29\theta \ldots \sin^2n\theta$	0	$+1$	Can be written as $0 \le \sin^2 n\theta \le 1$
$\cos^2\theta, \cos^23\theta, \cos^28\theta \ldots \cos^2n\theta$			$0 \le \cos^2 n\theta \le 1$
$\sin\theta \cos\theta$	$\frac{-1}{2}$	$\frac{+1}{2}$	$\frac{-1}{2} \le \sin\theta \cos\theta \le ½$

❖ **Few point to remember:**

1. **$\sin\theta$ and $\cos\theta$ both have "1" as their maximum value and "–1" as their minimum value. Hence, the values of sinx, sin2x, cosx, cos3x, etc. lie between –1 and 1.**

   $\sin\theta$ तथा **$\cos\theta$** दोनों का ही अधिकतम मान **"1"** तथा न्यूनतम मान **"–1"** होता है इसलिए **sin2x, cosx, cos3x,** इत्यादि के बाद **"–1"** तथा **"+1"** के बीच में स्थित होते हैं।

2. **For $\sin^2x$ and $\cos^2x$**
   - **Minimum value = 0**
   - **Maximum value = 1**

$Y = A + B$

$Y_{min} = A\downarrow + B\downarrow$

$Y_{max} = A\uparrow + B\uparrow$

$$Y = A - B$$

$$Y_{min} = A_{min} - B_{max}$$

$$Y_{max} = A_{max} - B_{min}$$

$Y = 3 - 2\sin^3\theta$

min^m = ↓ − ↑

= 3 − 2 = 1

max^m = ↑ − ↓

= 3 − (−2)

= 3 + 2

= 5

1. Find the min & max value of $3 - 2\sin^3\theta$

$3 - 2\sin^3\theta$ का न्यूनतम और अधिकतम मान ज्ञात कीजिए

(a) −1, 3 **(b) 1, 5**

(c) 1, 3 **(d) 0, 5**

$2\sin^3\theta$

$= 2(-1)$

$= -2$

TYPE-01

$$Y = a\sin\theta + b\cos\theta$$

Min: $-\sqrt{a^2+b^2}$ Max: $+\sqrt{a^2+b^2}$

$Y = \sin\theta + \cos\theta$

$Y = 2\sin\theta + 3\cos\theta$

$= a\sin\theta + b\cos\theta$

$Y = 3\sin\theta + 4\cos\theta$

$Y_{min} = -\sqrt{3^2+4^2} = -5$

$Y_{max} = \sqrt{3^2+4^2} = 5$

2. Find the minimum and maximum value of $3\sin\theta + 4\cos\theta$

$3\sin\theta + 4\cos\theta$ का न्यूनतम और अधिकतम मान ज्ञात कीजिए

(a) –5, 5
(b) 3, 4
(c) 0, 5
(d) –3, 4

$Y = 7\sin\theta - 24\cos\theta$

$\min = -\sqrt{7^2+24^2} = -25$

$\max = \sqrt{7^2+24^2} = 25$

3. Find the min & max value of the following expressions.

निम्नलिखित व्यंजकों का न्यूनतम और अधिकतम मान ज्ञात कीजिए।

(i) $7\sin\theta - 24\cos\theta$

(a) 7, 24

(b) –25, 25

(c) –7, 24

(d) –25, 0

$\min = -\sqrt{1+1} = -\sqrt{2}$

$\max = \sqrt{1+1} = \sqrt{2}$

(ii) $\sin\theta + \cos\theta$

(a) $-\sqrt{2}, \sqrt{2}$

(b) $-2, 2$

(c) $-1, 1$

(d) $0, \sqrt{2}$

$Y = 5\sin\theta + 12\cos\theta - 3$

$Y_{min} = -\sqrt{5^2+12^2} - 3$

$= -13 - 3 = -16$

$Y_{max} = +\sqrt{5^2+12^2} - 3$

$= 13 - 3$

$= 10$

(iii) $5\sin\theta + 12\cos\theta - 3$

(a) –16, 10

(b) –13, 13

(c) –16, 16

(d) 10, 13

4. Find minimum and maximum value of the following

निम्नलिखित का न्यूनतम और अधिकतम मान ज्ञात कीजिए

(i) $3\sin\theta + \cos\theta$

(a) $-\sqrt{10}, \sqrt{10}$ **(b) $-\sqrt{5}, \sqrt{5}$**

(c) 0, 3 **(d) $-\sqrt{3}, \sqrt{3}$**

$\text{min}^m = -\sqrt{3^2+1^2} = -\sqrt{10}$

$\text{max}^m = \sqrt{3^2+1^2} = \sqrt{10}$

(ii) $-7\sin\theta + 24\cos\theta$

(a) $-25, 25$ (b) $-7, 24$

(c) $7, 24$ (d) None of these

$$Y_{min} = -\sqrt{(-7)^2+(24)^2} = -25$$

$$Y_{max} = \sqrt{7^2+24^2} = 25$$

TYPE-02

$$Y = a\sin^2\theta + b\cos^2\theta \quad (a < b)$$

Min	Max	
a	b	(a < b)
b	a	(a > b)

$$Y = 3\sin^2\theta + 2\cos^2\theta$$

Maxm

$Y = 3\sin^2\theta + 2\cos^2\theta$

$= 3 \times 1 + 2 \times 0$

$= 3$

$\therefore$

Minm

$Y = 3\sin^2\theta + 2\cos^2\theta$

$\theta = 0°$

$= 0 + 2 \times 1$

$= 2$

5. **Find minimum and maximum value of**

का न्यूनतम और अधिकतम मान ज्ञात कीजिए

$$Y = 3\sin^2\theta + 4\cos^2\theta$$

(a) 3, 4
(b) 0, 3
(c) 0, 4
(d) None of these

6. Find minimum and maximum value of

का न्यूनतम और अधिकतम मान ज्ञात कीजिए

(i) $5\sin^2\theta + 4\cos^2\theta$

(a) 0, 4 **(b) None of these**

(c) 0, 5 **(d) 4, 5**

(ii) $3\sin^2\theta - 5\cos^2\theta$

(a) 3, 5 **(b) –5, 3**

(c) –3, 5 **(d) None of these**

$Y = 3\sin^2\theta - 5\cos^2\theta$

$Y_{min} = 0 - 5 = -5$

$Y_{max} = 3 - 0 = 3$

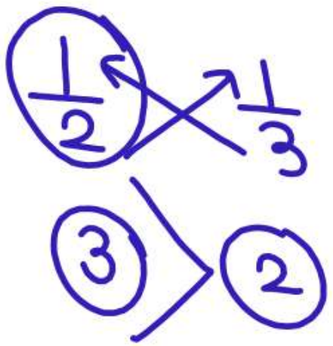
(iii) $\frac{1}{2}\sin^2\theta + \frac{1}{3}\cos^2\theta$

(a) $0, \frac{1}{3}$

(b) $-\frac{1}{3}, \frac{1}{2}$

(c) $\frac{1}{3}, \frac{1}{2}$

(d) None of these



(iv)– $5\sin^2\theta - 3\cos^2\theta$

(a) –5, –3 (b) 5, 3

(c) –5, 3 (d) 3, 5

$Y = -5\sin^2\theta - 3\cos^2\theta$

$Y_{min} = -5$ $Y_{max} = -3$

7. The minimum value of $2\sin^2\theta + 3\cos^2\theta$ is:

$2\sin^2\theta + 3\cos^2\theta$ का न्यूनतम मान है।

(a) 0
(b) 3
(c) 2
(d) 1

TYPE-03

$$Y = a\tan^2\theta + b\cot^2\theta \quad (a > 0, b > 0)$$

Min	Max
$2\sqrt{ab}$	∞

$Y_{min} = 2\sqrt{ab}$

$= 2\sqrt{4 \times 9}$

$= 2 \times 6 = 12$

8. **If $Y = 4\tan^2\theta + 9\cot^2\theta$ then find $Y_{min} = ?$**

यदि $Y = 4\tan^2\theta + 9\cot^2\theta$ है, तो Y का न्यूनतम मान ज्ञात कीजिए?

(a) 12 (b) 4

(c) 6 (d) 9

$Y_{min} = 2\sqrt{ab}$

$= 2\sqrt{8 \times 2}$

$= 2 \times 4$

$= 8$

9. If $Y = 8\tan^2\theta + 2\cot^2\theta$ then find Y_{min} = ?

यदि $Y = 8\tan^2\theta + 2\cot^2\theta$ है, तो Y का न्यूनतम मान ज्ञात कीजिए?

(a) 8 **(b) 4**

(c) 2 **(d) 6**

TYPE-04

$Y = a\sin^2\theta + b\,\mathrm{cosec}^2\theta \quad (a > 0, b > 0)$

$Y = a\cos^2\theta + b\sec^2\theta \quad (a > 0, b > 0)$

Min	Min	Max
$2\sqrt{ab}\,(a \geq b)$	$(a + b)\,(a \leq b)$	∞

10. **If $Y = 3\sin^2\theta + 12\,\mathrm{cosec}^2\theta$ then find Y_{min} = ?**

यदि $Y = 3\sin^2\theta + 12\,\mathrm{cosec}^2\theta$ है, तो Y का न्यूनतम मान ज्ञात कीजिए?

(a) 12 **(b) 15**

(c) 3 **(d) 9**

$a < b$

$\therefore Y_{min} = a + b$

$= 3 + 12$

$= 15$

11. If $Y = 12\sin^2\theta + 3\operatorname{cosec}^2\theta$ then find Y_{min} = ?

यदि $Y = 12\sin^2\theta + 3\operatorname{cosec}^2\theta$ है, तो Y का न्यनूतम मान ज्ञात कीजिए?

(a) 12 **(b) 15**

(c) 3 **(d) 9**

$a > b$

$Y_{min} = 2\sqrt{ab}$

$= 2\sqrt{12 \times 3}$

$= \underline{\underline{12}}$

12. If $Y = 8\cos^2\theta + 18\sec^2\theta$ then find Y_{min} = ?

यदि $Y = 8\cos^2\theta + 18\sec^2\theta$ है, तो Y का न्यनूतम मान ज्ञात कीजिए?

(a) 24
(b) 12
(c) 26 ✓
(d) 18

$a < b$

$Y_{min} = a + b$

$= 8 + 18$

$= \underline{\underline{26}}$

13. If $Y = 18\cos^2\theta + 8\sec^2\theta$ then find Y_{min} = ?

यदि $Y = 18\cos^2\theta + 8\sec^2\theta$ है, तो Y का न्यनूतम मान ज्ञात कीजिए?

(a) 24 (b) 12

(c) 26 (d) 18

$a > b$

$Y_{min} = 2\sqrt{ab}$

$= 2\sqrt{18 \times 8}$

$= 2 \times 12$

$= 24$

14. Find minimum value of the following.

निम्नलिखित का न्यूनतम मान ज्ञात कीजिए।

(i) $4\sin^2\theta + 9\,\text{cosec}^2\theta$

(a) 13 **(b) 4**

(c) 9 **(d) 5**

$a < b$

$Y_{min} = 4 + 9 = 13$

(ii) $8\cos^2\theta + 18\sec^2\theta$

(a) 8 **(b) 26**

(c) 18 **(d) 10**

$a < b$

$Y_{min} = 8 + 18$

$= 26$

(iii) $12 \sin^2\theta + 3 \operatorname{cosec}^2\theta$

(a) 3 **(b) 15**

(c) 12 **(d) 9**

$a > b$

$Y_{min} = 2\sqrt{12 \times 3}$

$= 12$

(iv) $4\sec^2\theta + 9\cos^2\theta$

(a) 4 **(b) 9**

(c) 5 **(d) 12**

$Y_{min} = 2\sqrt{4 \times 9}$

$= 12$

15. Find the minimum value of:

का न्यूनतम मान ज्ञात कीजिए:

(i) $32\cos^2\theta + 2\tan^2\theta$

(a) 4 **(b) 14**

(c) 2 **(d) None of these**

$= 32\cos^2\theta + 2(\sec^2\theta - 1)$

$= 32\cos^2\theta + 2\sec^2\theta - 2$

$Y_{min} = 8\sqrt{32 \times 2}$

$= 2 \times 8$

$= 16$

$ans = 16 - 2$

$= 14$

(ii) $4\sin^2\theta + 64\cot^2\theta$

(a) None of these **(b) 8**

(c) 4 **(d) 6**

$$4\sin^2\theta + 64(\operatorname{cosec}^2\theta - 1)$$

$$= \underbrace{4\sin^2\theta + 64\operatorname{cosec}^2\theta} - 64$$

$$4 + 64 - 64$$

$$= 4$$

(iii) $4\sec^2\theta + 9\operatorname{cosec}^2\theta$

(a) 4 (b) 9

(c) 6 (d) 25

$4\sec^2\theta + 9\operatorname{cosec}^2\theta$

$= 4(1+\tan^2\theta) + 9(1+\cot^2\theta)$

$= 4+4\tan^2\theta + 9 + 9\cot^2\theta$

$= 4\tan^2\theta + 9\cot^2\theta + 13$

$= 2\sqrt{4\times 9} + 13$

$= 12 + 13$

$= 25$

TYPE-05

$Y = a\sec^2\theta + b\,\mathrm{cosec}^2\theta \;(a > 0, b > 0)$

Min	Max
$\left(\sqrt{a}+\sqrt{b}\right)^2$	∞

16. $Y = 16\sec^2\theta + 25\,\text{cosec}^2\theta$ $\quad Y_{min} = ??$

(a) 81 (b) 64

(c) 49 (d) 36

$Y_{min} = (\sqrt{a}+\sqrt{b})^2$

$= (4+5)^2$

$= 81$

17. Find minimum value of the following

निम्नलिखित का न्यूनतम मान ज्ञात कीजिए

(a) $4\sec^2\theta + 25\operatorname{cosec}^2\theta$

(a) 36 (b) 49

(c) 25 (d) 16

$Y_{min} = (\sqrt{a}+\sqrt{b})^2$

$= (2+5)^2$

$= 49$

(b) $100\sec^2\theta + 9\,\mathrm{cosec}^2\theta$

(a) 144 (b) 121

(c) 169 (d) 189

$Y_{min} = (\sqrt{a} + \sqrt{b})^2$

$= (10 + 3)^2$

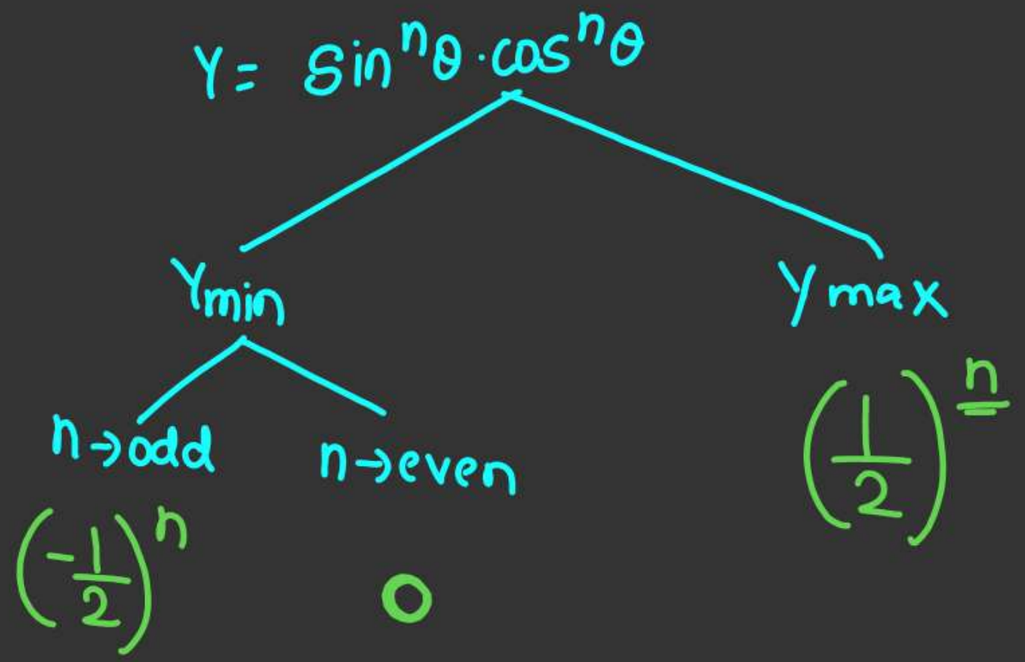
$= 169$

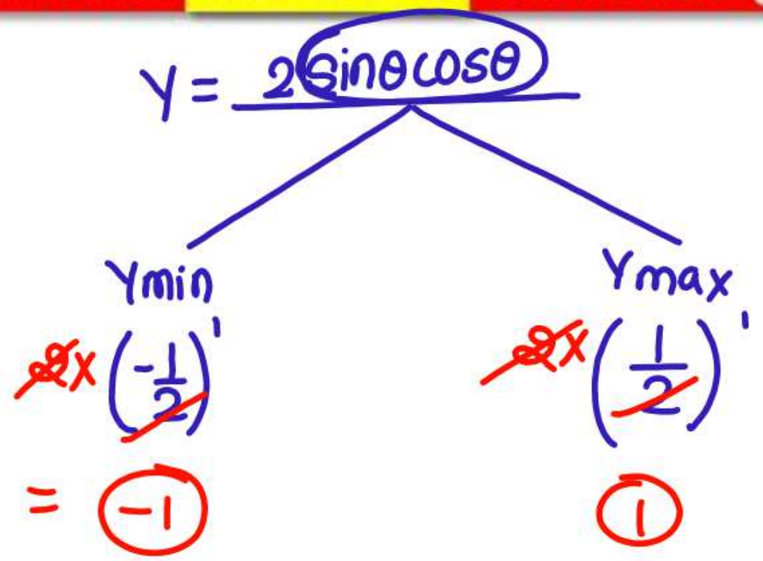
TYPE-06

$$\text{Min}(\sin^n\theta \cdot \cos^n\theta) = \left(\frac{-1}{2}\right)^n \quad , \quad (n \to \text{odd})$$

$$= 0 \quad , \quad (n \to \text{even})$$

$$\text{Max}(\sin^n\theta \cdot \cos^n\theta) = \left(\frac{1}{2}\right)^n$$


$Y = \sin^n\theta \cdot \cos^n\theta$
Y_{min}
Y_{max}
$n \to odd$
$n \to even$
$\left(\frac{-1}{2}\right)^n$
0
$\left(\frac{1}{2}\right)^n$



18. Find the minimum and Maximum value of 2 sinθ cosθ

2 sinθ cosθ का न्यूनतम और अधिकतम मान ज्ञात कीजिए

(a) 0, 1

(b) $-\frac{1}{2}, \frac{1}{2}$

(c) –1, 1

(d) None of these

$Y_{min} = \left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$

$Y_{max} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

19. Find the minimum and Maximum value of

$$Y = \sin^3\theta \cdot \cos^3\theta$$

$Y = \sin^3\theta \cdot \cos^3\theta$ का न्यूनतम और अधिकतम मान ज्ञात कीजिए

(a) $-\frac{1}{8}, \frac{1}{8}$

(b) $-\frac{1}{3}, \frac{1}{3}$

(c) $0, \frac{1}{8}$

(d) None of these

$A = \sin^1\theta \cdot \cos^1\theta$ $\quad A_{min} = \left(-\frac{1}{2}\right)^1$ $\quad A_{max} = \left(\frac{1}{2}\right)^1$

$B = \sin^2\theta \cos^2\theta$ $\quad B_{min} = 0$ $\quad B_{max} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$C = \sin^3\theta \cos^3\theta$ $\quad C_{min} = \left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$ $\quad C_{max} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$D = \sin^4\theta \cos^4\theta$ $\quad D_{min} = 0$ $\quad D_{max} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

20. The least value of $\cos\theta \sin\theta$ is:

$\cos\theta \sin\theta$ का न्यूनतम मान होगा।

SSC Phase X 05/08/2022 (Shift- 03)

(a) $\left(\frac{1}{2}\right)$ **(b) 0**

(c) (–1) **(d)** $\left(-\frac{1}{2}\right)$

$Y_{min} = 0$

$Y_{max} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

21. Find the minimum and Maximum value of

$$Y = \sin^4\theta \cdot \cos^4\theta$$

का न्यूनतम और अधिकतम मान ज्ञात कीजिए

(a) $0, \frac{1}{16}$ (b) $-\frac{1}{16}, \frac{1}{16}$

(c) None of these (d) $-\frac{1}{16}, 1$

$Y = 32 \sin^5\theta \cdot \cos^5\theta$

Y_{min} Y_{max}

$32 \times \left(-\frac{1}{2}\right)^5$ $32 \times \left(\frac{1}{2}\right)^5$

$= \cancel{32} \times \left(-\frac{1}{\cancel{32}}\right)$ $\cancel{32} \times \frac{1}{\cancel{32}}$

-1 1

22. Find the minimum and Maximum value of

$Y = 32 \sin^5\theta \cdot \cos^5\theta$

का न्यूनतम और अधिकतम मान ज्ञात कीजिए

(a) $-\frac{1}{32}, \frac{1}{32}$ **(b)** $-1, 1$

(c) 0, 1 **(d)** None of these

	Min	Max
* $a\sin\theta + b\cos\theta$	$-\sqrt{a^2+b^2}$	$\sqrt{a^2+b}$
* $a\sin^2\theta + b\cos^2\theta$	b	a
$a > b$	a	b
$a < b$		
* $a\tan^2\theta + b\cot^2\theta$	$2\sqrt{ab}$	∞
* $a\sin^2\theta + b\operatorname{cosec}^2\theta$ $a\cos^2\theta + b\sec^2\theta$		
$a > b$	$2\sqrt{ab}$	∞
$a < b$	$a+b$	∞
* $a\sec^2\theta + b\operatorname{cosec}^2\theta$	$(\sqrt{a}+\sqrt{b})^2$	∞

	min	max
$* \sin^n\theta \cdot \cos^n\theta$		
$n \to odd$	$\left(-\frac{1}{2}\right)^n$	$\left(\frac{1}{2}\right)^n$
$n \to even$	0	$\left(\frac{1}{2}\right)^n$

$\sin^m\theta + \cos^n\theta$	min	max
	$\theta = 45$	
Ex:- $\sin^3\theta + \cos^2\theta$	$\left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^2$	1
$\theta = 0$ or 90		

TYPE-07

$$\max(\sin^n\theta + \cos^m\theta) = 1 \qquad [m, n \geq 2]$$

$$(0 \leq \theta \leq 90)$$

23. Max($\sin^6\theta + \cos^{10}\theta$)

(a) 2 (b) 1

(c) 3 (d) 4

24. Max($\sin^{20}\theta + \cos^{40}\theta$)

(a) 1 (b) 2

(c) 0 (d) 3

25. The greatest value of $\sin^4\theta + \cos^4\theta$ is:

$\sin^4\theta + \cos^4\theta$ का अधिकतम मान है।

(a) 2 **(b) 3**

(c) $\frac{1}{2}$ **(d) 1**

M-1 $\theta = 45^\circ$

26. The minimum value of $\sin^2\theta + \cos^2\theta + \sec^2\theta + \text{cosec}^2\theta + \tan^2\theta + \cot^2\theta$ is:

$\sin^2\theta + \cos^2\theta + \sec^2\theta + \text{cosec}^2\theta + \tan^2\theta + \cot^2\theta$ का न्यूनतम मान है।

(a) 1 (b) 3

(c) 5 (d) 7

$\frac{1}{2} + \frac{1}{2} + 2 + 2 + 1 + 1$

$= 6 + 1$

$= 7$

M-2

26. The minimum value of $\sin^2\theta + \cos^2\theta + \sec^2\theta + \text{cosec}^2\theta + \tan^2\theta + \cot^2\theta$ is:

$\sin^2\theta + \cos^2\theta + \sec^2\theta + \text{cosec}^2\theta + \tan^2\theta + \cot^2\theta$ का न्यूनतम मान है।

(a) 1 (b) 3

(c) 5 (d) 7

$1 + 1 + \tan^2\theta + 1 + \cot^2\theta + \tan^2\theta + \cot^2\theta$

$= 3 + 2\tan^2\theta + 2\cot^2\theta$

$= 3 + 2(\tan^2\theta + \cot^2\theta)$

$= 3 + 2[2\sqrt{1 \times 1}]$

$= 3 + 4$

$= 7$

$A_{min} \rightarrow \theta = 45^\circ$

$A = \sin^2 45 + \cos^4 45$

$= \frac{1}{2} + \frac{1}{4}$

$= \frac{2+1}{4} = \frac{3}{4}$

$A_{max} \rightarrow 1$

27. If $A = \sin^2\theta + \cos^4\theta$, for any value of θ, then the value of A is:

यदि θ के किसी मान के लिए $A = \sin^2\theta + \cos^4\theta$ है तो A का मान है।

(a) $1 \le A \le 2$

(b) $\frac{3}{4} \le A \le 1$

(c) $\frac{13}{16} \le A \le 1$

(d) $\frac{3}{4} \le A \le \frac{13}{16}$

Min^m $\theta = 45^\circ$

$2 \times \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2$

$= 2 \times \left(\frac{1+2}{\sqrt{2}}\right)^2$

$= 2 \times \frac{9}{2}$

28. Find the minimum value of $(\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$.

$(\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$ का न्यूनतम मान कीजिए।

(a) 8 **(b) 7**

(c) 9 **(d) 4**

	min	max
* $\sin\theta+\cos\theta$	$-\sqrt{a^2+b^2}$ $=-\sqrt{1+1}$ $=-\sqrt{2}$	$\sqrt{a^2+b^2}$ $=\sqrt{1+1}$ $=\sqrt{2}$
* $\sin\alpha+\cos\beta$	$-1-1$ $=-2$	$1+1$ $=2$
* $\sin\alpha+\sin\beta+\sin\gamma$	-3	$+3$

$min^m = -7-24 = -31$

$Max^m = +7+24 = 31$

29. Find the maximum and minimum values of $7\cos\alpha + 24\sin\beta$.

$7\cos\alpha + 24\sin\beta$ का न्यूनतम और अधिकतम मान कीजिए।

(a) –7 and 7

(b) –25 and 25

(c) –24 and 24

(d) –31 and 31