QUADRATIC EQUATIONS

द्विघात समीकरण

PRACTICE SHEET

WITH SOLUTIONS

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Rankers Gurukul

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Quadratic Equation
(Practice Sheet With Solution)

1. If α and β are the roots of the equation $x^2 + x - 1 = 0$, what is the equation whose roots are α^5 and β^5 ?

यदि α तथा β समीकरण $x^2 + x - 1 = 0$ के मूल हैं, तो वह समीकरण क्या है जिसके मूल α^5 तथा β^5 हैं?

(a) $x^2 + 7x - 1 = 0$ (b) $x^2 - 7x - 1 = 0$

(c) $x^2 - 11x - 1 = 0$ (d) $x^2 + 11x - 1 = 0$

2. If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then which of the follow- ing is true?

यदि समीकरण $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ के मुल बराबर हैं, तो निम्नलिखित में से कौन सा सही है?

(a) $b = \frac{(a+c)}{ac}$ (b) $\frac{2}{b} = \left(\frac{1}{a}\right) + \left(\frac{1}{c}\right)$

(c) $2b = \left(\frac{1}{a}\right) + \left(\frac{1}{c}\right)$ (d) $abc = ab + bc + ca$

3. If the difference between the roots of the equation $Ax^2 - Bx + C = 0$ is 4, then which of the following is TRUE?

यदि समीकरण $Ax^2 - Bx + C = 0$ के मूलों का अंतर 4 है, तो निम्नलिखित में से कौन-सा सत्य है ?

(a) $B^2 - 16A^2 = 4AC + 4B^2$

(b) $B^2 - 10A^2 = 4AC + 6A^2$

(c) $B^2 - 8A^2 = 4AC + 10A^2$

(d) $B^2 - 16A^2 = 4AC + 8B^2$

4. α and β are the roots of quadratic equation. If $\alpha + \beta = 8$ and $\alpha - \beta = 2\sqrt{5}$, then which of the following equation will have roots α^4 and β^4?

α तथा β द्विघात समीकरण के मूल हैं। यदि $\alpha + \beta = 8$ तथा $\alpha - \beta = 2\sqrt{5}$ हैं, तो α^4 तथा β^4 निम्नलिखित में से किस समीकरण के मूल हैं ?

(a) $x^2 - 1522x + 14641 = 0$

(b) $x^2 + 1921x + 14641 = 0$

(c) $x^2 - 1764x + 14641 = 0$

(d) $x^2 + 2520x + 14641 = 0$

5. If a and b are the roots of the equation $Px^2 - Qx + R = 0$, then what is the value of $\left(\frac{1}{a^2}\right) + \left(\frac{1}{b^2}\right) + \left(\frac{a}{b}\right) + \left(\frac{b}{a}\right)$?

यदि a तथा b समीकरण $Px^2 - Qx + R = 0$ के मूल हैं, तो $\left(\frac{1}{a^2}\right) + \left(\frac{1}{b^2}\right) + \left(\frac{a}{b}\right) + \left(\frac{b}{a}\right)$? का मान क्या है?

(a) $\frac{(R - P)(Q^2 - 2RP)}{PR^2}$

(b) $\frac{(R + P)(Q^2 + 2RP)}{PR^2}$

(c) $\frac{(R + P)(Q^2 - 2RP)}{PR^2}$

(d) $\frac{(R + P)(P^2 - 2RQ)}{PR^2}$

6. α and β are the roots of the quadratic equation $x^2 - x - 1 = 0$. What is the value of $\alpha^8 + \beta^8$?

α तथा β द्विघात समीकरण $x^2 - x - 1 = 0$ के मूल हैं। $\alpha^8 + \beta^8$ का मान क्या है?

(a) 47 (b) 54

(c) 59 (d) 68

7. Sum of the roots of a quadratic equation is 5 less than the product of the roots. If one root is 1 more than the other root, find the product of the roots?

एक द्विघात समीकरण के मूलों का योग मूलों के गुणनफल से 5 कम है। यदि एक मूल दूसरे मूल से 1 अधिक है, तो मूलों का गुणनफल ज्ञात कीजिए?

(a) 6 or 3 (b) 12 or 2

(c) 8 or 4 (d) 12 or 4

8. If x, y, z are three factor of $a^3 - 7a - 6$ then value of $x + y + z$ will be

यदि x, y, z, $a^3 - 7a - 6$ के तीन गुणनखंड हैं तो $x + y + z$ का मान होगा

(a) 3a (b) 3b

(c) 6a (d) 9b

9. The value of a for which one root of the quadratic equation $(a^2 - 5a+3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is

a का मान जिसके लिए द्विघात समीकरण $(a^2 - 5a+3)x^2 + (3a - 1)x + 2 = 0$ का एक मूल दूसरे से दोगुना बड़ा है?

(a) $-2/3$ (b) $1/3$

(c) $-1/3$ (d) $2/3$

10. If (1 – p) is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its roots are
यदि (1 – p) द्विघात समीकरण $x^2 + px + (1 - p) = 0$ का मूल है तो इसके मूल हैं
(a) 0,–1 (b) –1,1
(c) 0,1 (d) –1,2

11. A complete factorisation of $x^4 + 64$ is
$x^4 + 64$ का पूर्ण गुणनखंडन है?
(a) $(x^2 + 8)^2$
(b) $(x^2 + 8)(x^2 - 8)$
(c) $(x^2 - 4x + 8)(x^2 - 4x + 8)$
(d) $(x^2 + 4x + 8)(x^2 - 4x + 8)$

12. $(x + 2)$ is a factor of $2x^3 + 5x^2 - x - k$. The value k is:
$(x + 2)$, $2x^3 + 5x^2 - x - k$ का एक गुणनखंड है। k का मान है:
(a) –24 (b) 6
(c) –6 (d) 24

13. The polynomial $4x^2 - kx + 7$ leaves a remainder of –2 when divided by $x - 3$. Find the value of k.
बहुपद $4x^2 - kx + 7$ को $x - 3$ से विभाजित करने पर शेषफल –2 बचता है। k का मान ज्ञात कीजिए।
(a) 17 (b) 19
(c) 15 (d) 23

14. If two polynomials $2x^3 + kx^2 + 4x - 12$ and $x^3 + x^2 - 2x + k$ leave the same remainder when divided by $(x - 3)$, find the value of k and the remainder.
यदि दो बहुपद $2x^3 + kx^2 + 4x - 12$ और $x^3 + x^2 - 2x + k$, से $(x - 3)$ से विभाजित करने पर समान शेषफल छोड़ते हैं, तो k का मान और शेषफल ज्ञात कीजिए।
(a) (–5, 29) (b) (7, 28)
(c) (–3,–27) (d) (– 3 ,27)

15. Find the value of m, if $x = 1/2$ is one of the zeroes of the polynomial $p(x) = 4x^4 - 4x^3 - mx^2 + 12x - 3$.
m का मान ज्ञात कीजिए यदि $x = 1/2$ बहुपद $p(x) = 4x^4 - 4x^3 - mx^2 + 12x - 3$ के शून्यकों में से एक है
(a) 9 (b) 11
(c) 7 (d) 13

16. Find the value of k if $p(x) = (3x - 2)(x - k) - 8$ is divided by $(x - 2)$ leaving the remainder 4.
k का मान ज्ञात कीजिए यदि $p(x) = (3x - 2)(x - k) - 8$ को $(x - 2)$ से विभाजित करने पर शेषफल 4 बचता है।
(a) –1 (b) –2
(c) +1 (d) +2

17. If $(x - 8)$ is one of the factors of $mx^3 - 24x^2 + 192x - 512$, find the value of m.
यदि $(x - 8)$, $mx^3 - 24x^2 + 192x - 512$ के गुणनखंडों में से एक है, तो m का मान ज्ञात कीजिए।
(a) 3 (b) 9
(c) 1 (d) 2

18. What should be subtracted from $x^4 + x^3 - 2x^2 + x + 1$ such that it is divisible by $x - 1$?
$x^4 + x^3 - 2x^2 + x + 1$ में से क्या घटाया जाना चाहिए ताकि यह $x - 1$ से विभाज्य हो जाए?
(a) 3 (b) 2
(c) 1
(d) More than one of the above

19. One of the linear factors of $3x^2 + 8x + 5$ is
$3x^2 + 8x + 5$ के रैखिक गुणनखंडों में से एक है।
(a) $(x + 1)$ (b) $(x - 4)$
(c) $(x - 2)$ (d) $(x + 2)$

20. For a polynomial p(x), p(–1) and p(2) are both equal to zero .So, we can conclude that,
एक बहुपद p(x) के लिए, p(–1) और p(2) दोनों शून्य के बराबर हैं। इसलिए, हम यह निष्कर्ष निकाल सकते हैं कि,
(a) $(x^2 + 2x - 1)$ is a factor
(b) $(x^2 - 2x + 1)$ is a factor
(c) $(x^2 - x - 2)$ is a factor
(d) $(x^2 - x + 2)$ is a factor

21. Find roots of the equation $4x^2 - 41x + 37$
समीकरण $4x^2 - 41x + 37$ के मूल ज्ञात कीजिए।
(a) $1, \frac{37}{4}$ (b) $2, \frac{27}{4}$
(c) $5, \frac{47}{4}$ (d) $6, \frac{37}{4}$

22. x – 2 is the HCF of the equation $4x^3 + 3x^2 - 8x - p = 0$ then find the value of P
समीकरण $4x^3 + 3x^2 - 8x - p = 0$ का मा.सा x – 2 है, तो P का मान ज्ञात करे।
(a) 24 (b) 17
(c) 28 (d) 31

23. What is the condition that the roots of the equation $ax^2 + bx + c = 0$ are in the ratio c : 1?
वह क्या प्रतिबंध है कि समीकरण $ax^2 + bx + c = 0$ के मूल c : 1 के अनुपात में हों?
(a) $b^2 = a(c + 1)^2$ (b) $a^2 = b(c + 1)^2$
(c) $b^2 = a(c - 1)^2$ (d) $b^2 = a(c + 2)^2$

24. $(x + 2)$ is a factor of $2x^3 + 5x^2 - x - k$. The value k is:
$(x + 2)$, $2x^3 + 5x^2 - x - k$ का एक गुणनखंड है। k का मान है:
(a) –24 (b) 6
(c) –6 (d) 24

25. If $x - y = 1$ and $x^2 + y^2 = 41$ where $x, y \geq 0$, then the value of $x + y$ will be:
यदि $x - y = 1$ और $x^2 + y^2 = 41$ है, जहाँ $x, y \geq 0$ है, तो $x + y$ का मान होगा।
(a) 9 (b) 8
(c) 6 (d) 7

Answer Key

1.(d)	**2.**(b)	**3.**(b)	**4.**(a)	**5.**(c)	**6.**(a)	**7.**(b)	**8.**(a)	**9.**(d)	**10.**(a)
11.(d)	**12.**(b)	**13.**(c)	**14.**(d)	**15.**(b)	**16.**(a)	**17.**(c)	**18.**(b)	**19.**(a)	**20.**(c)
21.(a)	**22.**(c)	**23.**(a)	**24.**(b)	**25.**(a)					

SOLUTIONS

1. (d)

$\alpha + \beta = -1, \alpha\beta = -1$

$\alpha^5 + \beta^5 = (\alpha^2 + \beta^2)(\alpha^3 + \beta^3) - \alpha^2\beta^2(\alpha + \beta)$

$\alpha^2 + \beta^2 = 1 - 2(-1) = 3$

$\alpha^3 + \beta^3 = -1 - 3(-1)(-1) = -4$

$\therefore \alpha^5 + \beta^5 = 3 \times (-4) - 1(-1) = -11$

$\therefore$ Required equation is $= x^2 - (-11)x - 1 = 0$

$\Rightarrow x^2 + 11x - 1 = 0$

2. (b)

$a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ roots are

We know

If roots are equal

Then,

$B^2 = 4AC$

$b^2(c - a)^2 = 4a(b - c) \times c(a - b)$

$b^2(c - a)^2 = 4ac(b - c)(a - b)$

On solving this :-

$2ac = ab + bc$

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

3. (b)

Let roots be α, β

$$\alpha + \beta = \frac{B}{A}$$

$\alpha - \beta = 4$

$$\alpha\beta = \frac{C}{A}$$

$$\alpha^2 + \beta^2 + 2\alpha\beta = \frac{B^2}{A^2} \quad ...(1)$$

$$\alpha^2 + \beta^2 - 2\alpha\beta = 16 \quad ...(2)$$

Subtracting (2) from (1), we get

$$4\alpha\beta = \frac{B^2}{A^2} - 16$$

$$4\frac{C}{A} = \frac{B^2 - 16A^2}{A^2}$$

$4CA = B^2 - 16A^2$

$B^2 - 10A^2 = 4CA + 6A^2$

4. (a)

$\alpha + \beta = 8, \alpha - \beta = 2\sqrt{5}$

Squaring :-

$\alpha^2 + \beta^2 + 2\alpha\beta = 64 \quad ...(1)$

$\alpha^2 + \beta^2 - 2\alpha\beta = 20 \quad ...(2)$

Subtracting (2) from (1), we get

$4\alpha\beta = 44$

$\alpha^2 + \beta^2 = 64 - 22 = 42$

Again squaring :-

$\alpha^4 + \beta^4 = 1764 - 2 \times 121 = 1522$

Qudratic eqn for root α^4 and β^4

$x^2 - (\alpha^4 + \beta^4) + (\alpha^4\beta^4) = 0$

$x^2 - 1522x + 14641 = 0$

5. (c)

$$a + b = \frac{Q}{P}, \qquad a.b = \frac{R}{P}$$

$$a^2 + b^2 + 2ab = \frac{Q^2}{P^2}$$

$$\Rightarrow \frac{a^2 + b^2}{ab} + \frac{2ab}{ab} = \frac{q^2}{p(ab)}$$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{a^2b^2} + \frac{a^2 + b^2}{ab}$$

$$\frac{a^2 + b^2}{ab}\left[\frac{1}{ab} + 1\right] = \frac{a^2 + b^2}{ab}\left\{\frac{1 + ab}{ab}\right\}$$

put value

$$\left[\frac{Q^2}{P^2ab} - \frac{2ab}{ab}\right]\left[\frac{1 + ab}{ab}\right]$$

$$\left[\frac{Q^2}{P^2 \times \frac{R}{P}} - 2\right]\left[\frac{1 + \frac{R}{P}}{\frac{R}{P}}\right]$$

$$= \frac{(R + P)(Q^2 - 2RP)}{PR^2}$$

6. (a)

$x^2 - x - 1 = 0$

$\alpha + \beta = 1, \alpha\beta = -1$

$\alpha^2 + \beta^2 + 2(-1) = 1$

$\alpha^2 + \beta^2 = 3, \alpha^4 + \beta^4 = 7$

$\alpha^8 + \beta^8 + 2(-1)^8 = 49$

$\alpha^8 + \beta^8 = 47$

7. (b)

Let,

roots of equation α, β

ATQ,

$\alpha\beta - (\alpha + \beta) = 5$ & $\alpha = \beta + 1$

$\beta(\beta + 1) - (\beta + \beta + 1) = 5$

$\beta^2 + \beta - 2\beta - 1 = 5$

$\beta^2 - \beta - 6 = 0$
$\beta^2 - 3\beta + 2\beta - 6 = 0$
$\beta(\beta - 3) + 2(\beta - 3) = 0$
$\beta = -2, 3$
then
$\alpha = 4, -1$
$\Rightarrow$ Required value
$\Rightarrow 4 \times 3, -1 \times -2 = 12, 2$

8. (a)
$a^3 - 7a - 6$
$a^2(a+1) - a(a+1) - 6(a+1)$
$=(a+1)(a^2 - a - 6)$
$(a+1)(a-3)(a+2)$
$a + 1 + a - 3 + a + 2 = 3a$

9. (d)
Let α, 2α are roots of given equation
Sum of roots

$$\alpha + 2\alpha = 3\alpha = \frac{1-3a}{a^2-5a+3} \quad ...(1)$$

And product of roots

$$\alpha(2\alpha) = 2\alpha^2 = \frac{2}{a^2-5a+3} \quad ...(2)$$

By (1) and (2), we have

$$\frac{9\alpha^2}{2\alpha^2} = \frac{(1-3a)^2}{(a^2-5a+3)^2} \times \frac{a^2-5a+3}{2}$$

$$9(a^2-5a+3) = (1-3a)^2$$

$$\Rightarrow a = \frac{2}{3}$$

10. (a)
$1 - p$ is root of $x^2 + px + 1 - p = 0$
then
$p = 1$
$x^2 + x + 0 = 0$
$x^2 + x = 0$
$x(x+1) = 0$
$x = 0, -1$

11. (d)
$x^4 + 64 = x^4 + 8^2$
$(x^2 + 8)^2 - 16x^2$
$= (x^2 + 4x + 8)(x^2 - 4x + 8)$

12. (b)
$f(x) = 2x^3 + 5x^2 - x - k$
$(x + 2)$ is factor.
$x = -2$ satisfies $f(x) = 0$
$f(-2) = -16 + 20 + 2 - k = 0$
$K = 6'$

13. (c)
let, $p(x) = 4x^2 - kx + 7$
given, -2 is Remainder when
$p(x)$ is divided by $x - 3$
So, $x = 3$ and $r = -2$
By remainder theorem:-
$p(x) = r$
$p(3) = -2$
$4(3)^2 - k(3) + 7 = -2$
$36 - 3k + 7 = -2$
$3k = 45$
$k = 15$

14. (d)
let $p(x) = 2x^3 + kx^2 + 4x - 12$
$f(x) = x^3 + x^2 - 2x + k$
given, $p(x)$ and $f(x)$ leaves same remainder when divided by $(x - 3)$
So, $x = 3$
$r = p(x) = f(x)$
then, $p(3) = f(3)$
$2(3)^3 + k(3)^2 + 4(3) - 12 = 3^3 + 3^2 - 2(3) + k$
$54 + 9k + 12 - 12 = 27 + 9 - 6 + k$
$54 + 9k = 30 + k$
$8k = -24$
$k = -3$
put $K = -3$ in $p(x)$,
$p(x) = 2x^3 + (-3)x^2 + 4x - 12$
$=2x^3 - 3x^2 + 4x - 12$
Now, $P(3) = 2(3)^3 - 3(3)^2 + 4(3) - 12$
$=54 - 27 + 12 - 12 = 27$
$K = -3$ and 27 remainder

15. (b)
$p(x) = 4x^4 - 4x^3 - mx^2 + 12x - 3$
$x = 1/2$ is zero of $p(x)$
So, $(x - 1/2)$ is a factor of equation)
By remainder theorem.
$r = p(a)$
$P(1/2) = 0$

$$4\left(\frac{1}{2}\right)^4 - 4\left(\frac{1}{2}\right)^3 - m\left(\frac{1}{2}\right)^2 + 12\left(\frac{1}{2}\right) - 3 = 0$$

$$\frac{4}{16} - \frac{4}{8} - \frac{m}{4} + 6 - 3 = 0$$

$$\frac{m}{4} = 3 - \frac{1}{4}$$

$$m = 11$$

16. (a)
$p(x) = (3x - 2)(x - k) - 8$
Also, it is given that the remainder is 4 when $p(x)$ is divided by $(x - 2)$

So, $x = 2$ and $r = 4$

Using remainder theorem:-

$p(x) = r$, $p(2) = 4$

$[3(2) - 2)(2 - k] - 8 = 4$

$(6 - 2)(2 - k) = 4 + 8$

$4(2 - k) = 12$

$2 - k = \frac{12}{4} = 3$

$k = 2 - 3 = -1$

17. (c)

let $p(x) = mx^3 - 24x^2 + 192x - 512$

given, $(x - 8)$ divides $p(x)$ exactly

Let $x = 8$ $r = 0$

By remainder theorem:-

$p(x) = r$

$P(8) = 0$

$m(8)^3 - 24(8)^2 + 192(8) - 512 = 0$

$512m - 1536 + 1536 - 512 = 0$

$512m = 512$

$m = 1$

18. (b)

$f(x) = x^4 + x^3 - 2x^2 + x + 1$

put $x - 1 = 0$ $x = 1$

let $f(x) = x^4 + x^3 - 2x^2 + x + 1$

$f(1) = 1 + 1 - 2 + 1 + 1 = 2$

Subtracting 2 from given

Equation makes it divisible by $(x - 1)$

19. (a)

$f(x) = 3x^2 + 8x + 5 = 0$

$= 3x^2 + 3x + 5x + 5 = 0$

$3x(x + 1) + 5(x + 1) = 0$

$(3x + 5)(x + 1) = 0$

$(x + 1)$ is one linear factor.

20. (c)

From options

$P(-1) = p(2)$

option (c)

$P(x) = x^2 - x - 2$

$P(-1) = 1 + 1 - 2 = 0$

$P(2) = 4 - 2 - 2 = 0$

$(x^2 - x - 2)$ is a factor

21. (a)

ATQ,

$\Rightarrow 4x^2 - 41x + 37$

$\Rightarrow 4x^2 - 37x - 4x + 37$

$\Rightarrow x(4x - 37) - 1(4x - 37)$

$\Rightarrow (x - 1)(4x - 37)$

$\Rightarrow x = 1, \frac{37}{4}$

22. (c)

ATQ

$\Rightarrow 4(2)^3 + 3(2)^2 - 8(2) - P = 0$

$\Rightarrow 32 + 12 - 16 - P = 0$

$\Rightarrow 28 - P = 0$

$\Rightarrow P = 28$

23.(a) Let α and β are the roots of given equation

$\frac{\alpha}{\beta} = \frac{c}{1}$ or $\frac{\beta}{\alpha} = \frac{1}{c}$

equation given, $ax^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{a}$ ... (1)

$\alpha\beta = \frac{c}{a}$

Squring (1)

$\alpha^2 + \beta^2 + 2\alpha\beta = \frac{b^2}{a^2}$

Divide this equation by $\alpha\beta$.

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2 = \frac{\frac{b^2}{a^2}}{\frac{c}{a}}$

$\Rightarrow c + \frac{1}{c} + 2 = \frac{b^2}{ac}$

$\Rightarrow a(c^2 + 1 + 2c) = b^2$

$a(c + 1)^2 = b^2$

24. (b)

$f(x) = 2x^3 + 5x^2 - x - k$

$(x + 2)$ is factor

$x = -2$ satisfied $f(x) = 0$

$f(-2) = -16 + 20 + 2 - k = 0$

$k = 6$

25. (a)

$(x + y) = \sqrt{(x - y)^2 + 4xy}$ .....(1)

Also,

$(x - y)^2 = x^2 + y^2 - 2xy$

$1 = 41 - 2xy$

$\Rightarrow xy = 20$ ....(2)

Put (2) in (1)

$(x + y) = \sqrt{1 + 80} = \sqrt{81} = 9$