TRIGONOMETRY

SHEET 02

Values of Trigonometric Ratios

(0°, 30°, 45°, 60° & 90°)

Trigonometry Table

	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

	0°	30°	45°	60°	90°
sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D
Coseco	N.D	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Secθ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	N.D
Cotθ	N.D	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

(a) $\sin 10^\circ < \sin 20^\circ$

(b) $\cos 10^\circ \geq \cos 20^\circ$

(c) $\sin 10^\circ < \cos 10^\circ$

(d) $\sin 70^\circ > \cos 70^\circ$

(e) $\sin 10^\circ < \sin 10$

* $\sin 10^\circ < \sin 20^\circ$

Note:- $\theta \uparrow$ $\sin\theta \uparrow$

* $\cos 10^\circ > \cos 20^\circ$

$\theta \uparrow$ $\cos\theta \downarrow$

Q. $0 < \theta < 45^\circ$ $\theta = 45^\circ$ $45^\circ < \theta \leq 90^\circ$

* $\cos\theta > \sin\theta$ $\cos\theta = \sin\theta$ $\boxed{\cos\theta < \sin\theta}$

$\sin 10^\circ < \cos 10^\circ$

$\sin 12^\circ < \cos 12^\circ$

$\sin 43^\circ < \cos 43^\circ$

$\sin 46^\circ > \cos 46^\circ$

$\sin 70^\circ > \cos 70^\circ$

$\sin 65^\circ > \cos 65^\circ$

θ → radian
θ → degree

$\cos 10$ → radian

$\cos 10^\circ$ → degree.

radian > degree

π (radian) = 180° (degree)

$\frac{22}{7}$ (rad) = 180° (degree)

1 rad = $\frac{180 \times 7}{22}$ (degree)

1 rad = 57 (degree)

1 m = 100 cm

$\sin 10^\circ \;\square\; \sin 10$

$\sin 10^\circ \;\boxed{<}\; \sin 570^\circ$

ⓐ $>$

ⓑ $<$

ⓒ $=$

ⓓ $\geq$

$3\sec^2 x - 4 = 0$

$\Rightarrow 3\sec^2 x = 4$

$\Rightarrow \sec^2 x = \frac{4}{3}$

$\Rightarrow \sec x = \frac{2}{\sqrt{3}}$

$x = 30^\circ$

1. **If $3\sec^2 x - 4 = 0$, then the value of x ? $(0 < x < 90^\circ)$ will be :**

CHSL 12/10/2020 (Afternoon)

(a) 15° **(b) 45°**

(c) 30° **(d) 60°**

$\sin\theta = \frac{1}{r}$

$\cos\theta = \frac{\sqrt{3}}{r}$

$\tan\theta = \frac{1}{\sqrt{3}}$

2. If $r\sin\theta = 1$, $r\cos\theta = \sqrt{3}$, then the value of $\left(\sqrt{3}\tan\theta + 1\right)$ is :

$1 \quad \sqrt{3} \times \frac{1}{\sqrt{3}} + 1$

(a) $\sqrt{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) 1

(d) 2

* $\frac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta$

* $\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$

M-1

$$\frac{2\times\frac{1}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2\times 3}{2\times\sqrt{3}} = \sqrt{3}$$

3. $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ} = ?$

SSC CGL 7 June 2019 (Evening)

(a) 3 (b) $\frac{1}{3}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

M-2

$= \tan 2\theta = \tan 60^\circ$

4. If $x \sin 60^\circ . \tan 30^\circ = \sec 60^\circ . \cot 45^\circ$, then the value of x is :

(a) $\sqrt{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) 4

(d) $4\sqrt{3}$

$x \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = 2 \times 1$

$x = 4$

5. If $x \sin 60^\circ . \tan 30^\circ - \tan^2 45^\circ = \text{cosec } 60^\circ . \cot 30^\circ - \sec^2 45^\circ$, then the value of x is :

(a) 2 (b) – 2

(c) 6 (d) – 4

$$x \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} - 1 = \frac{2}{\sqrt{3}} \cdot \sqrt{3} - 2$$

$$\frac{x}{2} - 1 = 0$$

$$\frac{x}{2} = 1 \Rightarrow x = 2$$

6. H.W

The numerical value of

$$\frac{\cos^2 45^\circ}{\sin^2 60^\circ} + \frac{\cos^2 60^\circ}{\sin^2 45^\circ} - \frac{\tan^2 30^\circ}{\cot^2 45^\circ} - \frac{\sin^2 30^\circ}{\cot^2 30^\circ}.$$

(a) $1\frac{1}{4}$

(b) $\frac{3}{4}$

(c) $\frac{1}{4}$

(d) $\frac{1}{2}$

7. H.W

The value of $\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin^2 90^\circ + 2\cos 90^\circ$ is :

SSC CGL 7 June 2019 (Evening)

(a) $\frac{15}{8}$

(b) $\frac{47}{24}$

(c) $\frac{23}{12}$

(d) 2

8. H.W

The value of

$$\frac{4\tan^2 30^\circ + \frac{1}{4}\sin^2 90^\circ + \frac{1}{8}\cot^2 60^\circ + \sin^2 30^\circ . \cos^2 45^\circ}{\sin 60 . \cos 30 - \cos 60 \sin 30}$$

is :

SSC CHSL 3 July 2019 (Evening)

(a) $1\frac{3}{4}$ (b) 4

(c) $2\frac{1}{2}$ (d) $3\frac{1}{2}$

9. H.W

$\frac{4}{3}\tan^2 60^\circ + 3\cos^2 30^\circ - 2\sec^2 30^\circ - \frac{3}{4}\cot^2 60^\circ$ is equal to :

SSC CPO 2018, 16 March 2019 (Evening)

(a) $\frac{8}{3}$ (b) $\frac{5}{4}$

(c) $\frac{10}{3}$ (d) $\frac{7}{3}$

10. H·W

The value of

$$\frac{\tan 30^\circ \operatorname{cosec} 60^\circ + \tan 60^\circ \sec 30^\circ}{\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ}$$ **is :**

SSC CGL 4 March 2020 (Morning)

(a) $\frac{2}{3}$

(b) $\frac{32}{3}$

(c) $\frac{8}{3}$

(d) $\frac{32}{99}$

$$= \frac{\cancel{4}^{2} \times \frac{1}{\cancel{2}} + 4}{\sqrt{3} \times 2 - 4 \times 1}$$

$$= \frac{6}{2\sqrt{3}-4} = \frac{3}{(\sqrt{3}-2)} \times \frac{(\sqrt{3}+2)}{(\sqrt{3}+2)}$$

$$= \frac{3(\sqrt{3}+2)}{(\sqrt{3})^2-(2)^2}$$

$$= -3(\sqrt{3}+2)$$

11. The value of

$$\frac{\text{cosec}^2 30^\circ \sin^2 45^\circ + \sec^2 60^\circ}{\tan 60^\circ \text{cosec}^2 45^\circ - \sec^2 60^\circ \tan 45^\circ}$$ **is :**

SSC CGL 2019 Tier-II (15/10/2020)

(a) $3\left(2+\sqrt{3}\right)$ (b) $2\left(\sqrt{3}-2\right)$

(c) $-2\sqrt{3}-2$ (d) $-3\left(2+\sqrt{3}\right)$

12. The value of

$$\frac{\sin^2 30^\circ + \cos^2 60^\circ + \sec 45^\circ \sin 45^\circ}{\sec 60^\circ + \operatorname{cosec} 30^\circ}$$ **is :**

CPO 24/11/2020 (Morning)

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$

(c) $-\frac{3}{8}$ (d) $\frac{3}{8}$

Rational alumber / (परिमेय संख्या)

↳ $\frac{p}{q}$ p, q are integer $q \neq 0$

$\frac{2}{3}$, $\frac{5}{7}$

Irrational Number (अपरिमेय संख्या)

$\frac{2}{\sqrt{3}}$, $\frac{\sqrt{5}}{7}$

13. Which among the following is an irrational quantity?

SSC CHSL 10 July 2019 (Afternoon)

(a) tan 30°.tan 60° **(b) sin 30°**

(c) tan 45° **(d) cos 30°**

(a) $\frac{1}{\sqrt{3}} \times \sqrt{3} = 1$

(b) $\frac{1}{2}$

(c) 1

(d) $\frac{\sqrt{3}}{2}$

$\sqrt{12} = \sqrt{2\times2\times3}$
$= 2\sqrt{3}$

14. If $\sin(x + 30°) = \frac{3}{\sqrt{12}}$, then the value of $x\ (0 < x < 90°)$ is :

SSC CHSL 26/10/2020 (Afternoon)

(a) 60° (b) 15°
(c) 45° (d) 30°

$\sin(x+30°) = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\sin(x+30°) = \sin 60°$

$x + 30° = 60°$

$x = 30°$

15. If $\sin\theta = \sqrt{3}\cos\theta$, $0° < \theta < 90°$, then the value of $2\sin^2\theta + \sec^2\theta + \sin\theta . \sec\theta + \text{cosec}\,\theta$ is :

SSC CGL 2019 Tier-II (11/09/2019)

(a) $\frac{33+10\sqrt{3}}{6}$ (b) $\frac{19+10\sqrt{3}}{6}$

(c) $\frac{33+10\sqrt{3}}{3}$ (d) $\frac{19+10\sqrt{3}}{3}$

$\frac{\sin\theta}{\cos\theta} = \sqrt{3}$

$\tan\theta = \sqrt{3}$

$\theta = 60°$

$2\sin^2 60° + \sec^2 60° + \sin 60° \cdot \sec 60° + \text{cosec}\, 60°$

$= \frac{2}{2}\times\frac{3}{4} + 4 + \frac{\sqrt{3}}{2}\times\frac{2}{1} + \frac{2\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$= \frac{3}{2} + \frac{4}{1} + \frac{\sqrt{3}}{1} + \frac{2\sqrt{3}}{3}$

$= \frac{9+24+6\sqrt{3}+4\sqrt{3}}{6} = \frac{33+10\sqrt{3}}{6}$

16. **If $3(\cot^2\varphi - \cos^2\varphi) = \cos^2\varphi$, $0° < \varphi < 90°$, then the value of $(\tan^2\varphi + \text{cosec}^2\varphi + \sin^2\varphi)$ is :**

SSC CGL Tier-II (12/09/2019)

(a) $\frac{13}{3}$ (b) $\frac{61}{12}$

(c) $\frac{25}{12}$ (d) $\frac{15}{4}$

17. If $0° < \theta < 90°$ and $\cos^2\theta = 3(\cot^2\theta - \cos^2\theta)$, then the value of $\left(\frac{1}{2}\sec\theta + \sin\theta\right)^{-1}$ is :

SSC CGL 4 June 2019 (Afternoon)

(a) $\sqrt{3}+2$ (b) $2(2-\sqrt{3})$

(c) $2(\sqrt{3}-1)$ (d) $\sqrt{3}+1$

$\cos^2\theta = 3\cot^2\theta - 3\cos^2\theta$

$\Rightarrow 4\cos^2\theta = 3\cot^2\theta$

$\Rightarrow 4\cos^2\theta = 3\frac{\cos^2\theta}{\sin^2\theta}$

$\Rightarrow \sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\theta = 60°$

$\left(\frac{1}{2}\sec 60° + \sin 60°\right)^{-1}$

$= \left(\frac{1}{2}\times 2 + \frac{\sqrt{3}}{2}\right)^{-1} = \left(\frac{2+\sqrt{3}}{2}\right)^{-1} = \frac{2}{2+\sqrt{3}} = 2(2-\sqrt{3})$

18. If $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$, $0° < \theta < 90°$, then the value of $\cot\theta + \operatorname{cosec}\theta$ is :

SSC CGL 6 June 2019 (Afternoon)

(a) $\sqrt{3}$

(b) $\frac{\sqrt{3}}{2}$

(c) $2\sqrt{3}$

(d) $\frac{3\sqrt{3}}{4}$

$\sin(A-B) = \sin 30^\circ$

$A - B = 30^\circ$

$A + B = 60^\circ$

$2A = 90^\circ$

$A = 45^\circ$

19. If $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$, where $A > B > 0°$, and $A + B$ is an acute angle, then the value of A is :

SSC CHSL 26/10/2020 (Afternoon)

(a) 45° (b) 30°

(c) 15° (d) 75°

$A+B=60^\circ$

$A-B=30^\circ$

$A=\frac{60+30}{2}$ $B=\frac{60-30}{2}$

$A=45^\circ$ $B=15^\circ$

$90^\circ+45^\circ=135^\circ$

20. If $\sin(\overset{60^\circ}{A+B})=\frac{\sqrt{3}}{2}$ and $\tan(\overset{30^\circ}{A-B})=\frac{1}{\sqrt{3}}$, then $(2A+3B)$ is equal to :

SSC CPO 2018, 13 March 2019 (Morning)

(a) 120° (b) 135°

(c) 130° (d) 125°

$\alpha = \frac{150}{2} = 75^\circ$

$\beta = \frac{30}{2} = 15^\circ$

21. **For α and β both being acute angles, it is given that $\sin(\alpha + \beta) = 1$, $\cos(\alpha - \beta) = \frac{1}{2}$. The values of α and β are :**

$\alpha + \beta = 90^\circ$, $\alpha - \beta = 60^\circ$

SSC CHSL 10 July 2019 (Evening)

(a) 75°, 15°
(b) 45°, 15°
(c) 75°, 45°
(d) 60°, 30°

$\frac{1}{\sqrt{2}}\times\frac{1}{2}\times\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\times\frac{2}{\sqrt{3}}+\frac{5\times1}{12\times1}$

22. The value of $\frac{1}{\sqrt{2}}\sin\frac{\pi}{6}\cos\frac{\pi}{4}-\cot\frac{\pi}{3}\sec\frac{\pi}{6}+\frac{5\tan\frac{\pi}{4}}{12\sin\frac{\pi}{2}}$ is equal to :

$\pi = 180°$

$= \frac{1}{4}-\frac{2}{3}+\frac{5}{12}$

$= \frac{3-8+5}{12} = 0$

(a) 0 ✓

(b) 1

(c) 2

(d) $\frac{3}{4}$

$\frac{\pi}{2} = \frac{180}{2} = 90°$

$\frac{\pi}{3} = \frac{180}{3} = 60°$

$\frac{\pi}{4} = \frac{180}{4} = 45°$

$\frac{\pi}{6} = \frac{180}{6} = 30°$

23. **The value of $152(\sin 30° + 2\cos^2 45° + 3 \sin 30° + 4\cos^2 45° + \ldots\ldots\ldots + 17 \sin 30° + 18 \cos^2 45°)$ is :**

(a) an integer but not a perfect square

(b) a rational number but not an integer

(c) a perfect square of an integer

(d) irrational

$$152\left(\frac{1}{2} + 2\times\frac{1}{2} + 3\times\frac{1}{2} + 4\times\frac{1}{2} + \ldots + 18\times\frac{1}{2}\right)$$

$$= \frac{152}{2}\left[1 + 2 + 3 + \ldots + 18\right]$$

$$= 76 \times \frac{18\times 19}{2}$$

$$= 19\times 2\times 18\times 19$$

$$= (19)^2\times(6)^2$$

$$= (114)^2$$

Sum of natural number $= \frac{n(n+1)}{2}$

24. If $x\cos60° + y\cos0° = 3$ and $4x\sin30° - y\cot45° = 2$, then what is the value of x?

(a) – 1 (b) 0

(c) 1 (d) 2

$\frac{x}{2} + y = 3$ — (i)

$2x - y = 2$ — (ii)

$\therefore$

$\frac{x}{2} + 2x = 5$

$\frac{5x}{2} = 5 \Rightarrow x = 2$

CGL 2017 Main
2018 "
2019 "
2020 "

25. Consider the following :

I. $\dfrac{\cot 30^\circ + 1}{\cot 30^\circ - 1} = 2(\cos 30^\circ + 1)$

II. $2\sin 45^\circ \cos 45^\circ - \tan 45^\circ \cot 45^\circ = 0$

Which of the above identities is/are correct?

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

(I) L.H.S $\dfrac{\sqrt{3}+1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{(\sqrt{3}+1)^2}{3-1} = \dfrac{4+2\sqrt{3}}{2}$

$= 2+\sqrt{3}$

R.H.S $= 2\left(\dfrac{\sqrt{3}}{2}+1\right) = 2\left(\dfrac{\sqrt{3}+2}{2}\right)$

(II) $2 \times \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} - 1 \times 1$

$= 1 - 1 = 0$