Geometric Progression (गुणोत्तर श्रेणी):

- **General form:** $a, ar, ar^2, ar^3, \ldots \ldots \ldots$

  $\times r \quad \times r \quad \times r$

  $AP \rightarrow T_{10} = a + 9d$

- **(n^{th} term)** $T_n = a \cdot r^{(n-1)}$

  $T_7 = ar^6$

  $\text{II} \longrightarrow 1 \text{ बार}$

- **(Sum of n terms)** $S_n = \dfrac{a(r^n - 1)}{r - 1} = \dfrac{a(1 - r^n)}{1 - r}$

  $\text{III} \longrightarrow 2 \text{ ,,}$

  $\text{IV} \longrightarrow 3 \text{ ,,}$

- **(Sum of infinite terms)** $S_\infty = \dfrac{a}{1-r}$, **(if** $-1 < r < 1$**)**

  $(\text{if } |r| < 1)$

Common ratio – सार्व अनुपात

1

$$20, \ -10, \ 5, \ -\frac{5}{2} \ \ldots \ldots$$

$a = 20$

$\gamma = -\frac{1}{2}$

→ Common ratio

सार्व अनुपात

$$a = 3 \qquad r = 2$$

$$3, 6, 12, 24 \quad .$$

$$T_8 = ar^7 = 3 \times 2^7$$

$$S_{10} = \frac{a(r^n - 1)}{r - 1} = \frac{3(2^{10} - 1)}{2 - 1}$$

$\boxed{\text{Decreasing GP}}$

$3, 6, 12, 24, \ldots \quad a=3, r=2$

$3, -6, 12, -24, 48, \ldots \quad a=3, r=-2$

$12, 6, 3, \frac{3}{2}, \ldots \quad a=12 \quad r=\frac{1}{2}$

$S_n = \frac{a(1-r^n)}{1-r} \qquad \left(\frac{1}{2}\right)^1 = \frac{1}{2}$

$S_\infty = \infty$

$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$S_\infty = \pm \infty$

$S_\infty = \frac{a(1-r^\infty)}{1-r} \qquad \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$S_\infty = -\infty$

$\boxed{S_\infty = \frac{a}{1-r}}$

$\left(\frac{1}{2}\right)^{1000} =$

$-3, -6, -12, -24, \ldots \quad a=-3$

$\left(\frac{1}{2}\right)^\infty = 0$

$\boxed{\text{Increasing GP}}$

$r = 2$

4

Geometric mean (गुणोत्तर माध्य):

- **Geometric mean of**

$$a_1, a_2, a_3, \ldots\ldots, a_n = \sqrt[n]{a_1 \times a_2 \times a_3 \times \ldots\ldots\ldots \times a_n}$$

a_1, b_1, c

$$= (a_1 \times a_2 \times a_3 \times \ldots \times a_n)^{\frac{1}{n}}$$

- **Geometric mean of** a, b **is** $\sqrt{ab}.$

$$= (ab)^{\frac{1}{2}}$$

- **If** $\underline{a}, b, c$ **are in GP then** $b^2 = ac$

$r = \dfrac{b}{a} = \dfrac{c}{b}$

$a_1 b_1 c \longrightarrow \sqrt[3]{a \times b \times c} = (abc)^{1/3}$

$b^2 = ac$

$a_1 b_1 c_1 d \longrightarrow (abcd)^{\frac{1}{4}} = \sqrt[4]{abcd}$

21. In a G.P., the 3rd term is 24 and the 6th term is 192. Find the 10th term.

एक गुणोत्तर श्रेणी में तीसरा पद 24 तथा 6वाँ पद 192 है, तो 10वाँ पद ज्ञात कीजिए।

a) 512   b) 1536

c) 3072   d) 6144

$$\frac{ar^2 = 24}{ar^5 = 192}$$

$$r = 2$$

$$\frac{1}{r^3} = \frac{1}{8}$$

$$r^3 = 8$$

$$r = 2$$

$$a = 6$$

$$T_{10} = a\, r^9$$

$$= 6 \times 512$$

$$= 3072$$

2^{12}

4^6

3^6

5^5

$$a r^4 = p$$

$$a r^7 = q$$

$$a r^{10} = s$$

$$p \times s \rightarrow a^2 r^{14}$$

$$q^2 = a^2 r^{14}$$

$$\boxed{q^2 = ps}$$

22. The 5th , 8th and 11th terms of a G.P. are p, q and s, respectively. Find q^2.

किसी गुणोत्तर श्रेणी का 5वीं, 8वाँ तथा 11वाँ पद क्रमशः p, q तथा s है तो दिखाइए कि $q^2 = ps$

a) $p + s$ b) p/s

c) ps d) s/p

$$(ar)^2 = ar^3$$

$$a^2r^2 = ar^3$$

$$a = r = -3$$

$$T_7 = ar^6 = (-3)^7 = -729 \times 3 =$$

The 4ᵗʰ term of a G.P. is square of its second term, and the first term is -3. Determine its. 7ᵗʰ term.

किसी गुणोत्तर श्रेणी का चौथा पद उसके दूसरे पद का वर्ग है तथा प्रथम पद -3 है तो 7वाँ पद ज्ञात कीजिए।

a) -2187 b) -343

c) -729 d) 343

$$a + ar + ar^2 = 16$$

$$\overline{ar^3 + ar^4 + ar^5} = 128$$

$$r^3(a + ar + ar^2)$$

$$\frac{1}{r^3} = \frac{1}{8}$$

$$\boxed{r = 2}$$

$$r = 2$$

$$7a = 16$$

$$a = \frac{16}{7}$$

24. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term.

किसी गुणोत्तर श्रेणी के प्रथम तीन पदों का योगफल 16 हैं तथा अगले तीन पदों का योग 128 है। प्रथम पद ज्ञात करे।

a) 2   b) 16

c) $\frac{16}{7}$ d) $\frac{7}{16}$

10

$$\frac{ar^2 - a = 9}{ar - ar^3 = 18}$$

$r = -2$

$$\frac{a(r^2 + 1)}{ar(1 - r^2)} = -\frac{9}{18}$$

$r = -2$

$3a = 9$

$\boxed{a = 3}$

25. **Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4th by 18.**

ऐसे चार पद ज्ञात कीजिए जो गुणोत्तर श्रेणी में हो, जिसका तीसरा पद प्रथम पद से 9 अधिक हो तथा दूसरा पद चौथे पद से 18 अधिक हो।

$3, -6, 12, -24$

$$a(r-1) = 12$$
$$ar - a = 12$$
$$ar^2 - ar = 28$$

$$\frac{a(r-1)}{ar(r-1)} = \frac{12}{28} \cdot \frac{3}{7}$$

$$\boxed{r = \frac{7}{3}}$$

$$ar^3 - ar^2$$
$$= ar^2(r-1)$$

$$\overset{4}{12} \times \frac{7}{3} \times \frac{7}{3} = \frac{196}{3} = 65\frac{1}{3}$$

26. **The second term of a geometric progression is 12 more than the first term and the third term is 28 more than the second. What is the difference between the third and fourth terms of that progression?**

एक गुणोत्तर श्रेणी का दूसरा पद पहले पद से 12 अधिक है और तीसरा पद दूसरे से 28 अधिक है। उस श्रेणी के तीसरे और चौथे पद के बीच क्या अंतर है?

a) 44

b) 65.33

c) 60

d) 80

The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio.

एक गुणोत्तर श्रेणी के तीन पदों का योगफल $\frac{39}{10}$ गुणोत्तर हैं तथा उनका गणनफल 1 है। सार्व अनुपात ज्ञात कीजिए।

a) $\frac{5}{2}$ b) $\frac{2}{5}$

c) $\frac{5}{2}$ or $\frac{2}{5}$ d) 1

$$a + ar + ar^2 = \frac{39}{10}$$

$$a^3 r^3 = 1$$

$$a = \frac{1}{r}$$

$$\boxed{ar = 1}$$

$$\boxed{a = \frac{1}{r}}$$

$$r = \frac{5}{2}$$

$$\frac{2}{5}, 1, \frac{5}{2}$$

$$r = \frac{2}{5}$$

$$a = \frac{5}{2}, 1, \frac{2}{5}$$

$$\frac{1}{r} + 1 + r = \frac{39}{10}$$

$$\frac{1}{r} + r = \frac{29}{10}$$

$$\frac{1 + r^2}{r} = \frac{29}{10}$$

$$10r^2 - 29r + 10 = 0$$

Prod = 100

Sum = 29

$$\frac{25}{10} = \frac{5}{2}$$

$$\frac{4}{10} = \frac{2}{5}$$

13

$\boxed{AP} \longrightarrow$ $a-d,\ a,\ a+d$

$\longrightarrow a-2d,\ a-d,\ a,\ a+d,\ a+2d$

$\longrightarrow a-3d,\ a-d,\ a+d,\ a+3d$

$\boxed{GP} \longrightarrow \dfrac{a}{r},\ a,\ ar$

$\longrightarrow \dfrac{a}{r^2},\ \dfrac{a}{r},\ a,\ ar,\ ar^2$

$\longrightarrow \dfrac{a}{r^3},\ \dfrac{a}{r},\ ar,\ ar^3$

14

$$\frac{a}{r} \times a \times ar = 1$$

$$a^3 = 1$$
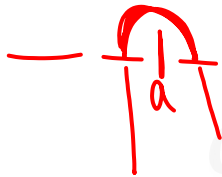
$$\boxed{a = 1}$$

$a = 1$

$$\frac{1}{r} + 1 + r = \frac{39}{10}$$

$$\frac{1 + r^2}{r} = \frac{29}{10}$$

$$a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$$

$$a^{\frac{1}{x}} = k \implies a = k^x$$

$$b^{\frac{1}{y}} = k \implies b = k^y$$

$$c^{\frac{1}{z}} = k \implies c = k^z$$

$$b^2 = ac$$

$$k^{2y} = k^x \cdot k^z$$

$$\implies k^{2y} = k^{x+z}$$

$$2y = x+z$$

$$x, y, z \text{ AP}$$

$b^2 = ac$

28. If a, b, c are in G.P. and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, then x, y, z are in

यदि a, b, c गुणोत्तर श्रेणी में हैं तथा $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, हैं तो x, y, z _____ श्रेणी में है।

a) AP   b) GP

c) HP   d) None

16

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow \quad \overset{63}{\cancel{315}} = \frac{\cancel{5}(2^n - 1)}{2 - 1}$$

$$\Rightarrow \quad 2^n = 64 = 2^6$$

$$n = 6$$

$$T_6 = ar^5 = 5 \times 32 = 160$$

29. The sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2, respectively. Find the last term.

गुणोत्तर श्रेणी के कुछ पदों का योग 315 है, उसका प्रथम पद तथा सार्व अनुपात क्रमशः 5 तथा 2 हैं। अंतिम पद ज्ञात कीजिए।

a) 80
b) 160
c) 180
d) 320

$$\frac{a+b}{2} = 12.5$$

$$\Rightarrow a+b = 25$$

$$ab = 144 <\genfrac{}{}{0pt}{}{16}{9} \quad diff = 7$$

$$\sqrt{ab} = 12$$

30. If the arithmetic and geometric mean of two numbers are 12.5 and 12 respectively, find the difference of the two numbers?

यदि दो संख्याओं का समांतर और गुणोत्तर माध्य क्रमशः 12.5 और 12 हैं, तो दोनों संख्याओं का अंतर ज्ञात कीजिए?

a) 0.5 b) 5
c) 6   d) 7

18

31. If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

यदि किसी द्विघात समीकरण के मूलों के समांतर माध्य एवं गुणोत्तर माध्य क्रमशः 8 तथा 5 हैं, तो द्विघात समीकरण ज्ञात कीजिए।

a, b

$$a + b = 16$$

$$ab = 25$$

$$x^2 - (SoR)x + PoR = 0$$

$$\Rightarrow x^2 - 16x + 25 = 0$$

P, q, r, s

$1, 2, 4, 8$

$a = 1$

$r = 2$

$(1+4+16)(4+16+64)$

$= 21 \times 84$

$\underset{2}{\underbrace{}}$

If p, q, r, s are in geometric progression, then $\left(p^2 + q^2 + r^2\right)\left(q^2 + r^2 + s^2\right) =$

यदि p, q, r, s गुणोत्तर श्रेणी में हैं, तो $\left(p^2 + q^2 + r^2\right)\left(q^2 + r^2 + s^2\right) =$

a) $pqrs$

b) $p + q + r + s$

c) $(pq + qr + rs)^2$

d) $p^2 + q^2 + r^2 + s^2$

$(2 + 8 + 32)^2$

$= 42 \times 42$

p, q, r, s

$1, 2, 4, 8$

$(1+8), (8+64), (64+512)$

$9, 72, 576$

$\times 8 \quad \times 8$

33. If p, q, r, s are in geometric progression, then $(p^3 + q^3), (q^3 + r^3), (r^3 + s^3)$ are in:

यदि p, q, r, s गुणोत्तर श्रेणी में हैं, तो $(p^3 + q^3), (q^3 + r^3), (r^3 + s^3)$ निम्न में हैं:

a) A.P
b) G.P
c) H.P
d) None

$$1 + 2 + 3 + 4 + \ldots + n = \sum n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \quad . \quad + n(n+1) = \sum n(n+1) = \frac{n(n+1)(n+2)}{3}$$

$$1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \quad + n(n+1)(n+2)$$

$$= \sum n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

Eg: $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + 10 \times 11 = \frac{10 \times 11 \times 12}{3} = 440$

n

34. **Find the sum of the first 40 terms of**
$$2 + 4 + 3 + 9 + 4 + 16 + \ldots \ldots$$
$2 + 4 + 3 + 9 + 4 + 16 + \cdots$ के पहले
40 पदों का योग ज्ञात कीजिए

a) 2048 b) 3310
c) 3540 d) 3542

$$2 + 4 + 3 + 9 + 4 + 16 + 5 \ldots$$

$$2 + 3 + 4 + \quad + 21$$
$$2^2 + 3^2 + 4^2 + \ldots + 21^2$$

$$\left[\begin{array}{l} 1 + 2 + 3 + \quad + 21 \\ 1^2 + 2^2 + 3^2 + \ldots + 21^2 \end{array} \right] - 2 = Ans$$

$$\frac{21 \times 22}{2} + \frac{21 \times 22 + 43}{6} - 2 =$$

$$\frac{21 \times 22}{2} \left[1 + \frac{43}{3} \right] - 2$$

$$= \frac{\overset{7}{\cancel{21}} \times \overset{11}{\cancel{22}} \times 46}{\cancel{2} \times \cancel{3}} - 2$$

$$= 322$$

$$3542 - 2 = 3540$$

Harmonic Progression (हरात्मक श्रेणी):

- **General form:** $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}, \ldots \ldots \ldots$

- **(n^{th} term)** $T_n = \frac{1}{a+(n-1)d}$

$$AP \rightarrow T_n = a + (n-1)d$$

$$AP \longrightarrow 7, 10, 13, 16, \ldots$$

$$HP \rightarrow \frac{1}{7}, \frac{1}{10}, \frac{1}{13}, \frac{1}{16}, \ldots$$

$$AP \rightarrow \frac{3}{2}, 2, \frac{5}{2}, 3, \ldots$$

$$HP \rightarrow \frac{2}{3}, \frac{1}{2}, \frac{2}{5}, \frac{1}{3}, \quad .$$

Harmonic mean (हरात्मक माध्य):

- **Harmonic mean of**

$$a_1, a_2, a_3, \ldots\ldots, a_n = \frac{n}{\dfrac{1}{a_1} + \dfrac{1}{a_2} + \dfrac{1}{a_3} + \cdots\ldots + \dfrac{1}{a_n}}$$

- **Harmonic mean of $\underline{a, b}$ is** $\dfrac{2}{\left(\dfrac{1}{a} + \dfrac{1}{b}\right)} = \dfrac{2ab}{a+b}.$

$$AM = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

$$\frac{2}{\frac{1}{a} + \frac{1}{b}} = \frac{2}{\frac{a+b}{ab}} = \frac{2ab}{a+b}$$

$$\frac{AM}{HM} = \frac{\frac{a+b}{2}}{\frac{2}{\frac{1}{a}+\frac{1}{b}}} = \frac{(a+b)\times(a+b)}{2\times 2ab} = \frac{25}{9}$$

a, b

$$\boxed{\frac{2ab}{a+b}}$$

Square root

$$\frac{AM}{GM} = \frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{a+b}{2\sqrt{ab}} = \frac{5}{3}$$

35. There are two numbers. If the ratio of their arithmetic mean to harmonic mean is $25:9$, what is the ratio of their arithmetic mean to geometric mean?

दो संख्याएं हैं. यदि उनके समांतर माध्य का हरात्मक माध्य से अनुपात $25:9$ है, तो उनके समांतर माध्य का गुणोत्तर माध्य से अनुपात क्या है?

a) $3:5$ b) $5:3$

c) $9:25$ d) $25:9$