Basic Terms used in T.O.C

① Symbol :-

Any character

Building block of this SubJect

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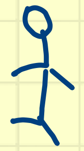
a, b, c, d ..... A, B, C .. @, $ 

0 1 2 ---- 9

② Alphabet (Σ) :-

Non empty finite set of symbols

$\Sigma = \{a, b\}$

$\Sigma = \{a, b, c, A, 0\}$

$\Sigma = \{A, B, C, D, 1, 2, 3, 0, \$\}$

$\Sigma = \{0, 1, 2, 3 \ldots\ldots\}$ ✗

$\Sigma = \{\}$ ✗

③ String :-

Finite sequence of symbols from Σ.

sequence → Order matter

$\Sigma = \{a, b\}$

$S_1 = aabbaab$ ✓ $|S_1| = 7$

$S_2 = ab$ ✓

$S_3 = aaa$ ✓ → a^3

$S_4 = bbbbb$ ✓ → b^5

$S_5 = aabbcd$ ✗

$S_6 = ba$

$S_7 = aaabbb\ldots$ ✗

$\Sigma = \{a, b, c, 0, 1, \$\}$

$S_1 = ab01\$abc$ ✓

Aho Ulman ✓
Peter linz → Basic
Daniel I A Cohen
Mihal Sipser ✓
Papadhimitriou → TM

Null string :-

$\Sigma = \{0, 1, 2, 3\}$

$|S_1| = 3$

$S_1 \Rightarrow 2\,2\,2 \Rightarrow 2^3$

$|S_2| = 5$

$S_2 \Rightarrow 2\,2\,2\,2\,2 \Rightarrow 2^5$

$|S_3| = 2$

$S_3 = 2^2 \Rightarrow 2\,2$

$|S_4| = 1$

$S_4 = 2^1 \Rightarrow 2$

$|S_5| = 0$

$S_5 = 2^0 = \epsilon$

$S_6 = 0^0 = \epsilon$

$S_7 = 0^3 \Rightarrow 000$

$S_8 = 1^0 \Rightarrow \epsilon$

$1^3 = 111$

$2^3 = 8$

↳ $2\,2\,2$

$2^5 = 32$

$2^0 = 1$

$2^0 = 0$

$\epsilon \rightarrow$ Absence of Symbol

④ Length of String :-

Number of symbols in the string

Length of Nullstring => 0

$\Sigma = \{a, b\}$

$\Sigma = \{a, b, c, d, e, f\}$

$S = abcd$

Q

$\Sigma = \{ab, ba\}$

$S_1 = abba$

$|S_1| = 2$

$S_2 = abbab$

$|S_2| =$

Invalid String

Q Length of Null string? ⇒ 0

⑤ Substring :-

0 or more contineous symbols taken from string.

$\Sigma = \{a, b, c, d\}$

$S = abcd$

$S_1 = \epsilon$ ✓

$S_2 = a$ ✓

$S_3 = ab$ ✓

$S_4 = ac$ ✗

$S_5 = acd$ ✗

$S_6 = abc$ ✓

$S_7 = bc$ ✓

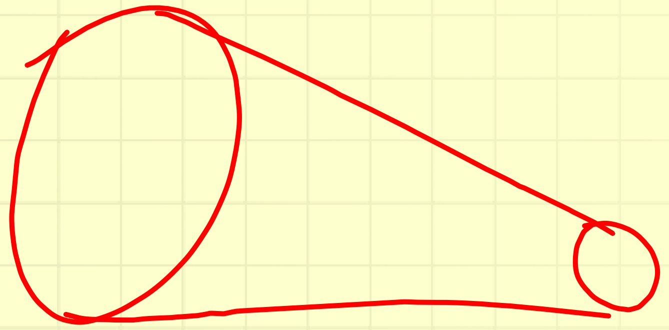
$S_8 = abcd$ ✓

Trivial substrings

$S_1 = \epsilon$

$S = \epsilon$ ✓

ϵ string is substring of every string



Symbol → Alphabet → String → Language

6) Language :-

Set of strings

$\Sigma = \{a, b\}$ → Strings = ∞ → Set of strings → Subsets → ∞

$L = \{ab, ba, aabb, aaa, abb\}$

Set → ⊂ , ⊆

Language → ⊂ , ⊆

Language of strings over a & b which starts with ends with a

$L = \{a, aa, ba, aaa, aba, baa, bba\}$

⇒ Language of strings over a & b which starts with a

$L = \{a, aa, ab, aaa, aab, aba, abb, aaaa, aaab, \ldots\}$

$\Sigma = \{a, b\}$

1 2 3 4 1 2 6 7 8 9 ....

How many Strings ?? → ∞

Length 0 → ϵ ⇒ 1

1 → a, b ⇒ 2

2 → aa, ab, ba, bb ⇒ 4

3 → $2 * 2 * 2 = 2^3 = 8$

4 → 2 2 2 2 ⇒ $2^4 = 16$

⋮

∞ → ∞

Natural no.

{1 - - - to ∞}

$\Sigma = \{0, 1, 2, 3, 4, \ldots 9\}$

many ✗

$S = \epsilon$

$S_1 = 12347621$ ✓

$S_2 = 0$

$|S_3| = 1$

$\infty \in N$ ✗

$\infty \in R$ ✗

$\infty \in C$ ✗

→ $\Sigma = \{0, 1\}$

No. of strings will be ∞ on Σ but each & every string will be of finite length

$S \Rightarrow 2^S$

$L = \phi$

$L = \{ \}$

Language which contain No string at all

Empty Language

ϕ → Empty set

ϵ → Empty String

$L = \{\epsilon\}$ → Language which contain empty string as an element

Set of all Strings over Σ → Σ^* ⇒ Universal Language

$\Sigma = \{a, b\}$

$\Sigma^* \Rightarrow \{\epsilon, a, b, aa, ab, ba, bb, aaa, aab, \ldots\}$

$L \subseteq \Sigma^*$

2^{Σ^*} → Set of all languages over Σ

Language → set of strings

String → finite sequence of symbols taken from Σ

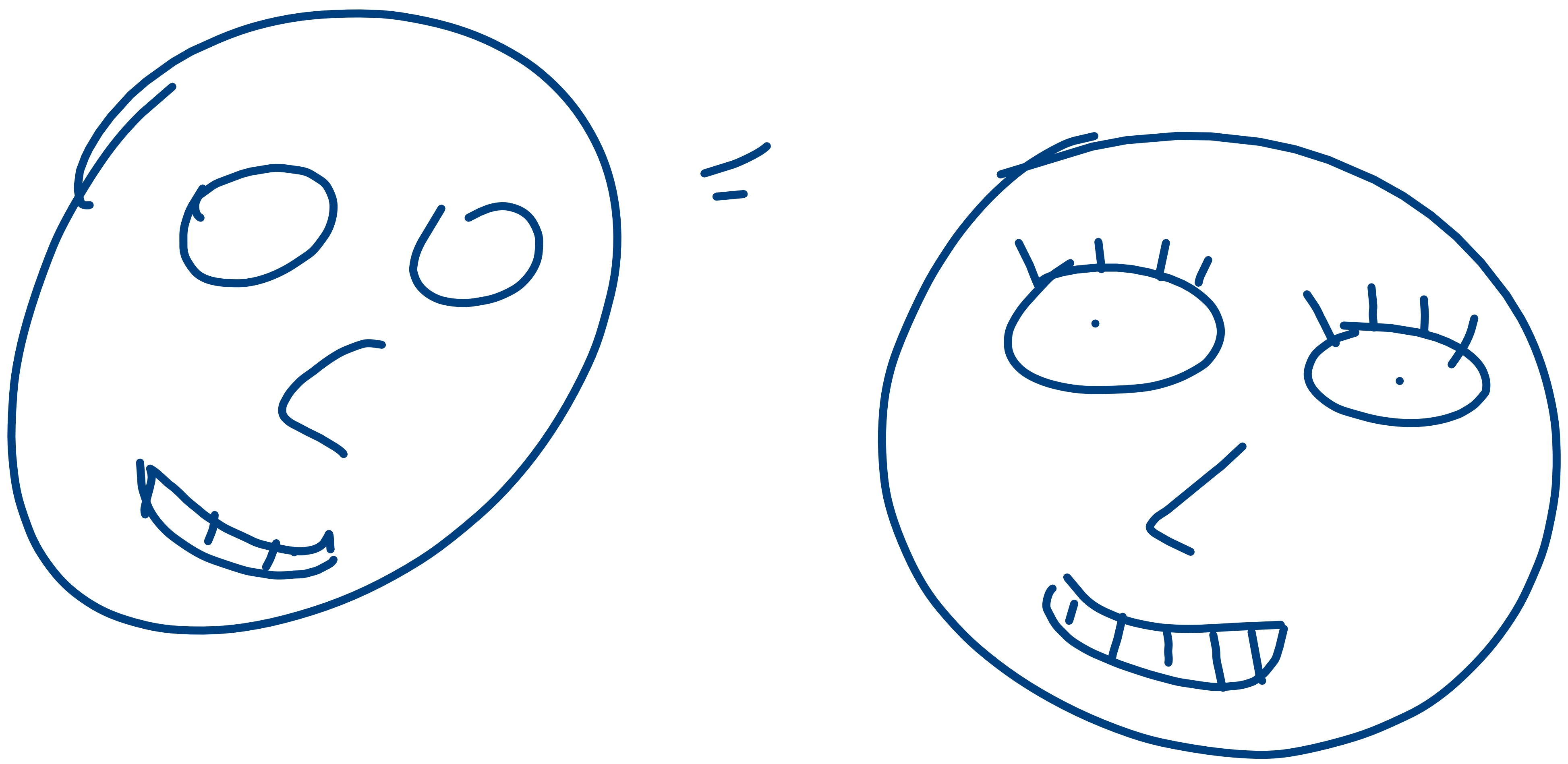
Empty string ⇒ ϵ

Empty Language ⇒ $\{ \}$ ⇒ ϕ

$\{\epsilon\}$ ≠

ϕ ↓ $\{ \}$

$\{\phi\}$ ↓ $\{\{ \}\}$



Universal Language :-

$\Sigma = \{a, b\}$

$\Sigma^* = \{\epsilon, a, b, aa, ab, ba, bb, aaa \ldots\}$

$\Sigma = \{\ \}$ X

$\Sigma^* = \{$

$\Sigma = \{a\}$

$\Sigma^* = \{\epsilon, a, aa, aaa, aaaa, aaaaa \ldots\}$

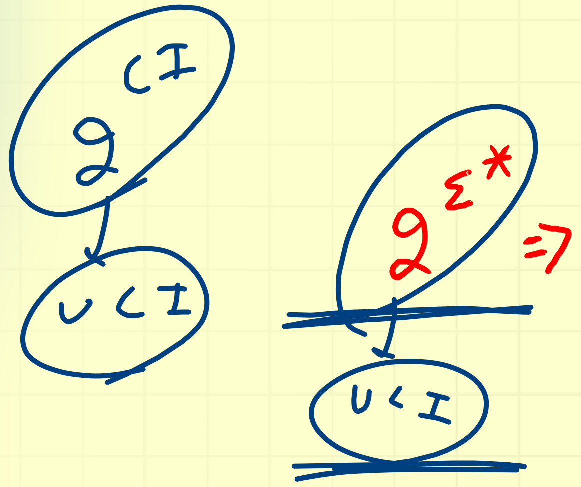
$\Sigma = \{a, b, c\}$

$\Sigma^* = \{\epsilon, a, b, c, aa, bb, cc, ab, ac, bc, ba \ldots\}$

2 length

Σ^* Σ = a

aa
ab
ac
ba
bb
bc
ca
cb
cc



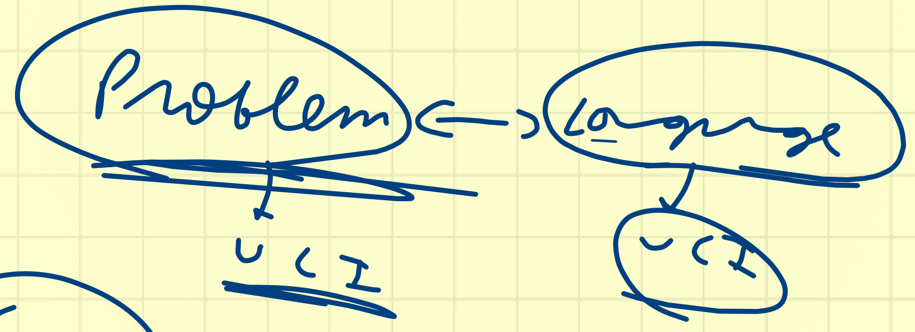
CI → $\Sigma = \{a, b\}$

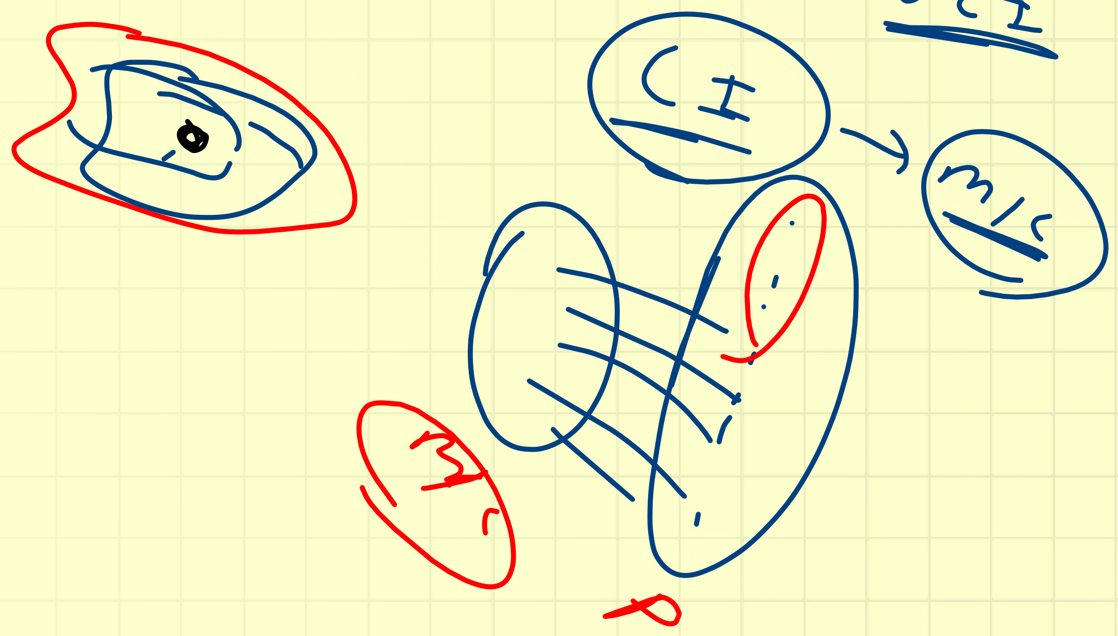
$\Sigma^* = \{\epsilon, a, b, \ldots\}$

$\{a, b\}$

$\{ab, ba\}$

2^{Σ^*} ⇒ Set of all languages possible over Σ





Symbol ✓

↓

Alphabet ✓

↓

String ✓

↓

Language ✓

The number of substrings (of all lengths inclusive) that can be formed from a string of length n *whose all characters are diffrent is*

a) n

b) n^2

c) $n(n-1)/2$

d) $n(n+1)/2+1$

GATE 1994

$\Sigma = \{a, b, c\}$

$S = abc$

$0 \rightarrow \epsilon \rightarrow 1$ _

$1 \rightarrow \{a, b, c\} \rightarrow 3$ _

$2 \rightarrow ab, bc \rightarrow 2$ _

$3 \rightarrow abc \rightarrow 1$ _

$\frac{1}{3}$

$\Sigma = \{a_1, a_2, a_3 \ldots a_n\}$

$S = a_1 a_2 a_3 a_4 \ldots a_n$

$0 \rightarrow \epsilon \Rightarrow 1$

$1 \rightarrow a_1, a_2, \ldots, a_n \Rightarrow n - 0$

$2 \Rightarrow a_1 a_2, a_2 a_3, a_3 a_4, \ldots, a_{n-1} a_n \Rightarrow n-1$

$3 \Rightarrow a_1 a_2 a_3, a_2 a_3 a_4, \ldots, a_{n-2} a_{n-1} a_n \Rightarrow n-2$

$4 \Rightarrow n-3$

$\vdots$

$n \Rightarrow n-(n-1)$

$\Rightarrow 1 \ldots$

a b c (d) ??

$\frac{n(n+1)}{2} + 1$

a b c d (e) (f)

$1 + 2 + 3 + 4 \ldots n$

$\frac{n(n+1)}{2}$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{n}{2}(2 + (n-1))$

$= \frac{n}{2}(2 + n - 1)$

$= \frac{n(n+1)}{2}$

String itself included

$$\frac{n(n+1)}{2} + 1 - 1$$

$$= \frac{n(n+1)}{2}$$

Trivial substring exclude

$$\frac{n(n+1)}{2} + 1 - 2$$

$$\frac{n(n+1)}{2} - 1$$

ϵ excluded

$$\frac{n(n+1)}{2} + 1 - 1$$

$$\frac{n(n+1)}{2}$$

Q How many substrings of length 'k' ?

0 → 1

1 → n - 0

2 → n - 1

3 → n - 2

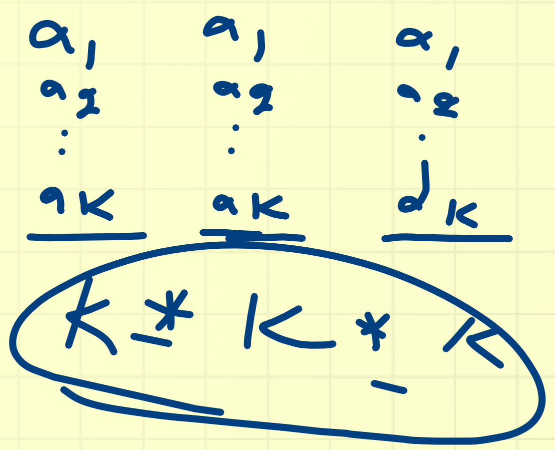
⋮

k → n - (k - 1)

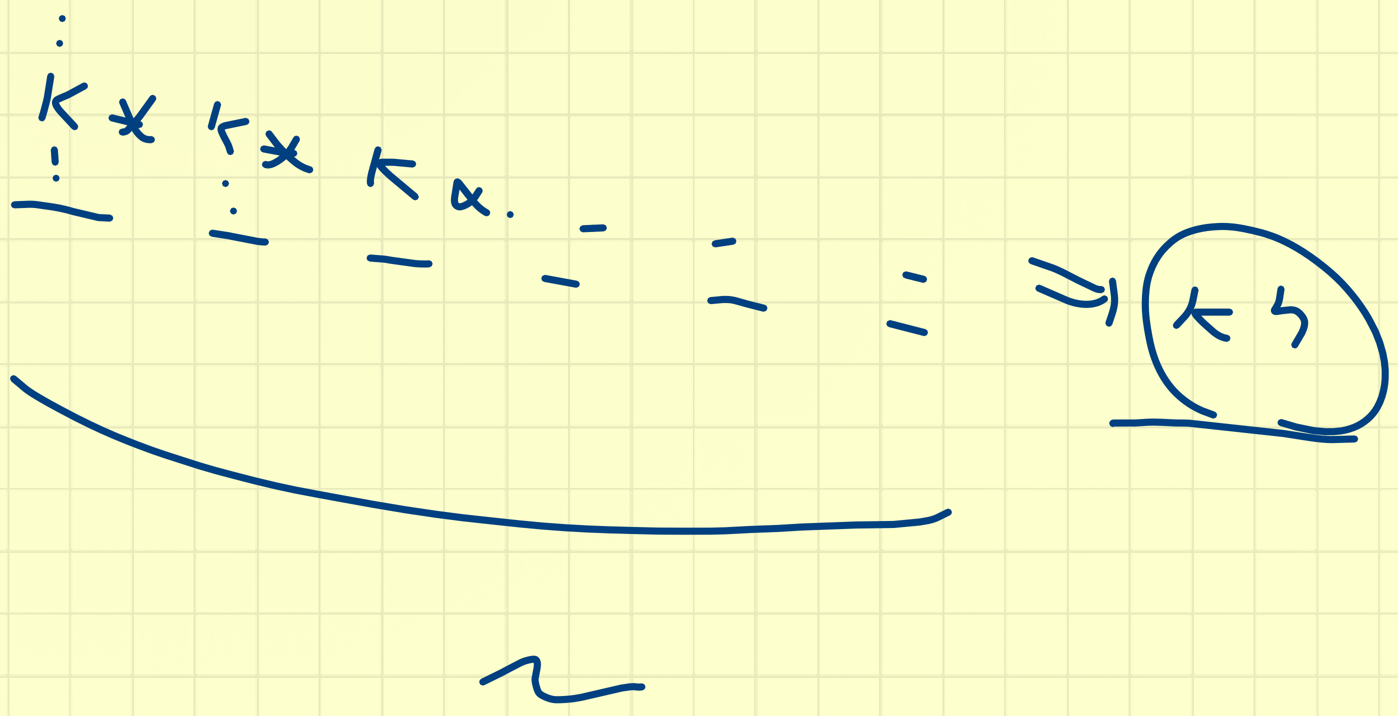
n - k + 1

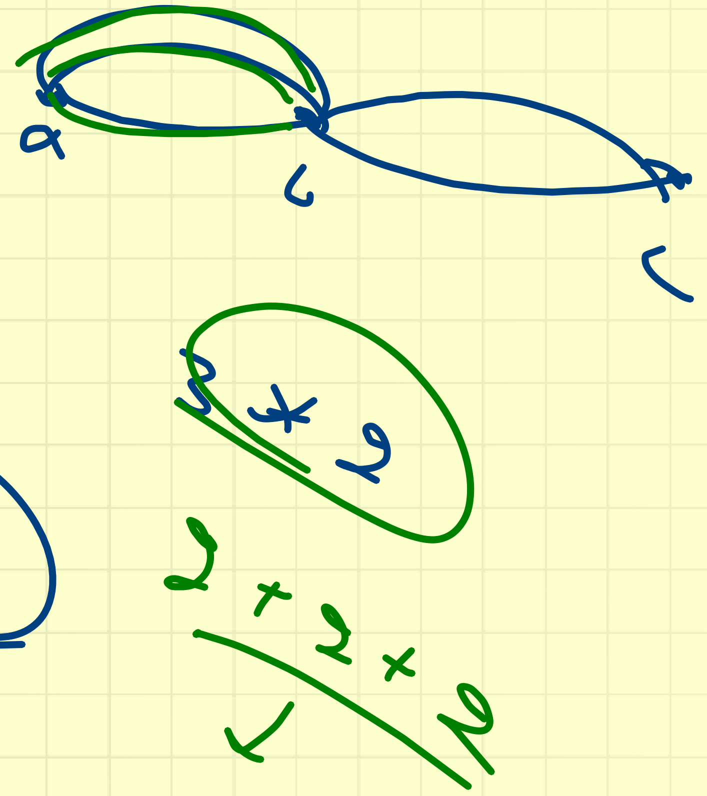
The number of substrings (of all lengths inclusive) that can be formed from a string of length *n whose all characters are SAME is*

$\Sigma = \{a_1, a_2, a_3 \, --- \, a_k\}$



The number of strings of length N that can be formed using K symbols





$|\Sigma| \Rightarrow$

$\Sigma = \{a_1, a_2 \ldots a_k\}$

The number of strings of length N

$|\Sigma| = k$

$\underbrace{k \times k \ldots}_{n}$ k^n

$\underbrace{|\Sigma| * |\Sigma| * |\Sigma| \ldots}_{n} \Rightarrow (|\Sigma|)^n$

atmost 'n'

$|\Sigma|^n + |\Sigma|^{n+1} \ldots |\Sigma|^{\infty}$ ✗

∞ =

The number of strings of length atmost N

$L_0 + L_1 + L_2 \ldots L_n$ (n+1)

$|\Sigma|^0 + |\Sigma|^1 + |\Sigma|^2 \ldots |\Sigma|^n$

No. of terms

$S_n = a * \left(\frac{r^n - 1}{r - 1}\right)$

a: 1st term

$= 1 * \left(\frac{|\Sigma|^{n+1} - 1}{|\Sigma| - 1}\right)$

Q || No. of strings over $\Sigma = \{a, b\}$ will be $\underline{\infty}$ & each string will be of $\underline{\text{finite}}$ length

$\Sigma = \{0 \ldots 9\}$

Powers of Σ

$\Sigma = \{a, b\}$

$\Sigma^0 = \{\epsilon\}$

$\Sigma^1 = \{a, b\}$

$\Sigma^2 = \{aa, ab, ba, bb\}$

$\Sigma^3 = \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$

$\Sigma^4 = \{ \_\_\_ \quad \_\_\_\_\_\_ \}$

$\Sigma \times \Sigma$

$\{a, b\} \times \{a, b\}$

$\{(a, a), (a, b), (b, a), (b, b)\}$

$$\Sigma = \{a, b\}$$

$$\Sigma^* = \{\epsilon, a, b, aa, ab, ba, bb, aaa, aab \dots\}$$

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \dots$$

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \Sigma^4 \dots$$

$$\Sigma^+ = \Sigma^* - \Sigma^0 \qquad \Sigma^* - \{\epsilon\}$$

$$\Sigma^+ = \Sigma^* - \{\epsilon\}$$

$$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \dots$$